Calculation of Theoretical Torque and Displacement in an Internal Gear Pump

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This paper describes numerical determination of theoretical torque (ideal torque) and theoretical stroke volume (pump displacement) in an internal gear pump without crescent. The design analyzed has been commonly used for an automatic transmission and continuously variable transmission in vehicles because of its high mechanical efficiency. For estimating the pump efficiencies accurately, determining the accurate theoretical torque and theoretical displacement of the pump is important.

This paper presents a calculation method that takes into consideration the contact points for meshing and sealing between the driving and driven gear for the accurate theoretical torque and displacement of an internal gear pump without crescent such as a gerotor pump.

Key Words: *hydraulic power system, hydraulic pump, internal gear pump, theoretical torque, theoretical displacement, calculation method*

1. Introduction

Internal gear pumps used in relatively low pressure conditions have been widely used for automatic transmissions and engines to supply oil for both lubrication and hydraulic control. They are used because of their simple structure with fewer components and easy assembly than other options. Currently, they have been used in CVT (Continuous Variable Transmission) with operating pressures over 5 MPa[1), 2)]. In the meantime, to improve fuel economy in vehicles, auxiliary components including hydraulic pumps are required to increase their efficiencies. Currently a conventional internal gear pump has a crescent placed between the tips of the driving and driven gears to separate the delivery pressure side and the suction pressure side and to prevent oil leakage from the delivery pressure side. Now this has been replaced with an internal gear pump without crescent. In this design the sealing is performed by the tooth tips of the driving and driven gears. For improving the efficiencies of the pump, it is important to determine accurately the theoretical torque and the displacement volume of the pump.

Although for the internal gear pump with crescent including an involute gear pump the calculation method for theoretical torque and displacement has already been proposed[3)], it has yet to be clarified for the internal gear pump without crescent.

This paper presents the calculation method for the theoretical torque and the theoretical displacement for internal gear pump without crescent, by using a similar calculation method to a balanced vane pump, which has been previously reported[4)].

2. Nomenclature

b : Gear width
F_1 : Force due to pressure acting on tooth face with meshing point of driven gear
F_2 : Force received from driving gear on meshing point of driven gear
F_3 : Force due to pressure acting on tooth face with sealing point of driven gear
F_4 : Force due to pressure acting on tooth face with meshing point of driving gear
F_5 : Force received from driven gear on meshing point of driving gear
F_6 : Force due to pressure acting on tooth face with sealing point of driving gear
N : Pump speed
p_d : Delivery pressure
p_s : Suction pressure
Δp : Difference between delivery and suction pressure ($= p_d - p_s$)
r_1 : Radius from center of driven gear to meshing point
r_2 : Radius from center of driven gear to sealing point
r_3 : Radius from center of driving gear to meshing point
r_4 : Radius from center of driving gear to sealing point
T_{th} : Theoretical torque
V_{th} : Theoretical displacement per revolution
z : Number of teeth on driving gear

ϕ : Pressure angle of tooth at meshing point
θ : Rotational angle of driving gear
Subscript:
R : Component in radial direction
θ : Component in circumferential direction

3. Structure of the Pump

First, the internal gear pump without crescent is described below. The pump is composed of a driving gear (an external tooth gear placed inside) and a driven gear (an internal tooth gear placed outside). Various tooth profiles are in use, among which the trochoidal profile and the combined profile with hypocycloidal curves shown in **Fig. 1** are well known. Unlike the pump with crescent to prevent leakage between the tips of the driving and driven gears, shown in **Fig. 2**, these pumps without crescent are designed so the tooth tip parts of both gears come in contact at one point to separate suction part from delivery part (in practice a very small clearance is provided to avoid locking of the gears).

In contrast to the internal gear pump with crescent, the internal gear pump without crescent has a larger volume between the teeth of the driving and driven gears. This means that the pump without crescent can have a larger theoretical displacement when the tooth width, the tip diameter of the driving gear and the outer diameter of the driven gear are the same as those of the pump with crescent. In other words, when the tooth width and the tip diameter of the driving gear are the same, the outer diameter of the driven gear in the pump without crescent can be smaller with the same displacement. Especially at high-speed pump operation, it can decrease the friction torque caused by shearing viscous oil on the sliding parts of the gears, improving the mechanical efficiency. That is the reason why this type of gear pump has often been used in the automotive automatic transmissions, including CVTs.

In the gear pump, the meshing point between the driving and driven gears changes depending on the rotating position (angle) of the driving gear. As shown in **Fig. 3**, the teeth of both the driving gear and the driven gear are subject to the suction pressure and/or the delivery pressure. The tooth faces meshing with each other receive different pressures separated at the meshing point. Also, the tooth faces concerned with sealing receive different pressures separated at the sealing point. In addition, for the gear pump without crescent the sealing point is not always on the tip of the tooth, as shown in **Fig. 4**. It changes depending on the rotational angle of the driving gear. This difference from the internal gear pump with crescent makes the accurate calculation of the theoretical torque and the displacement difficult.

In the gear pump, the meshing points of the driving gear and the driven gear transfer to the next teeth during rotation. The locus of the meshing point as well as the sealing point changes in the radial direction.

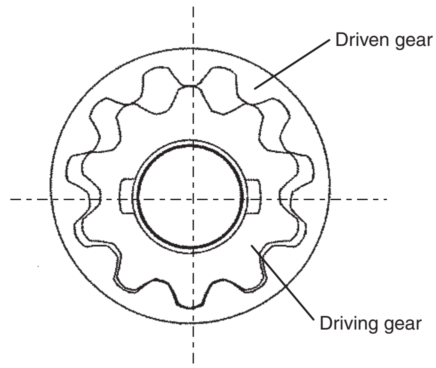

Fig. 1 Configuration of internal gear pump without crescent

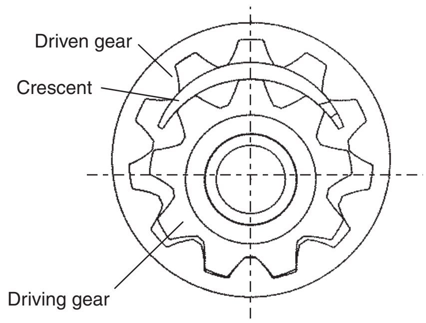

Fig. 2 Configuration of internal gear pump with crescent

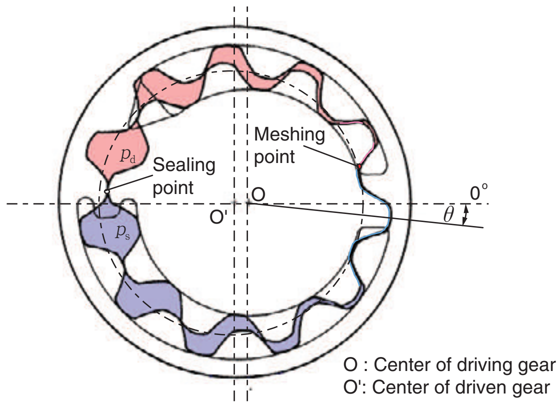

Fig. 3 Pressure acting in internal gear pump

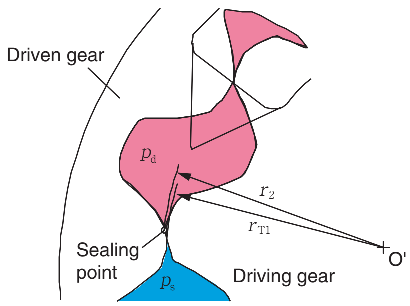

Fig. 4 Tip circle radius and seal radius of driven gear

4. Calculation of Theoretical Torque and Displacement

4. 1 Calculation Method

There are two calculation methods for the theoretical displacement. One is to calculate geometrically the volume of the space enclosed by the teeth of the driving and driven gears. The other method first requires calculation of the theoretical torque of the pump. Then the displacement of the pump should be calculated by using the theoretical torque. This is based on the principle that the hydraulic energy given to the oil by the pump operation equals the power required to drive the pump shaft, disregarding energy loss. The former approach used to have a problem with accuracy because of the integral of the volume between the teeth of the driving and driven gears. In recent years, thanks to the spread of CAD, the calculation has become relatively easier. Nevertheless, it still requires drawings of the driving and driven gears for each minute change of rotational angle. To avoid this trouble with accurate calculation when calculating the theoretical torque, the latter method was selected and described below.

In this calculation method, the torque to rotate the driving gear is considered against the difference between the suction pressure and the delivery pressure. These pressures act on the tooth face in the direction perpendicular to it. However, for the calculation of the theoretical torque, only the pressure acting in the circumferential direction around the center of rotation must be considered. For the teeth without the meshing point or the sealing point, the force due to the pressure does not need to be considered. As shown in **Fig. 5**, the force due to the pressure acts on both right and left faces of the tooth evenly. In the case of the internal gear pump, it is necessary only to consider the force caused by the pressure difference on the teeth with the meshing point and the sealing point.

First the force and the torque working on the driven gear are considered. The relationship between the pressure and the force working on the driven gear is shown in **Fig. 6**. Both the suction pressure p_s and the delivery pressure p_d act on some of the teeth of the driven gear. The difference between p_d and p_s is denoted as Δp. The force due to the pressure $\vec{F}_1$ acts on the tooth with the meshing point and the force due to the pressure $\vec{F}_3$ acts on the sealing point in the vicinity of the tooth tip in the driven gear. Then, the driven gear receives the reaction force $\vec{F}_2$ and the circumferential component against the sum of $\vec{F}_1$ and $\vec{F}_2$ are at the meshing point from the driving gear. In the actual pump, the theoretical torque to drive the pump depends only on the component of the force in the circumferential direction (θ). The values of $\vec{F}_1$ and $\vec{F}_3$ are known, and they can substitute for each equivalent concentrated force as expressed below:

$$|\vec{F}_{1\theta}| = \Delta pb(r_1 - r_{T1}) \quad (1)$$

$$|\vec{F}_{3\theta}| = \Delta pb(r_2 - r_{T1}) \quad (2)$$

The radii r_{m1} and r_{m2} from the center of the driven gear O' to the points acting $\vec{F}_1$ and $\vec{F}_3$ are expressed by the following equations, respectively.

$$r_{m1} = (r_1 + r_{T1})/2 \quad (3)$$

$$r_{m2} = (r_2 + r_{T1})/2 \quad (4)$$

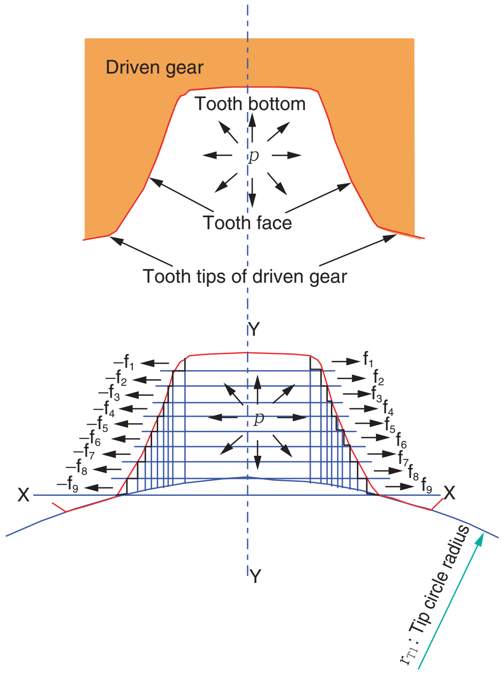

Fig. 5 Pressure and force acting on tooth

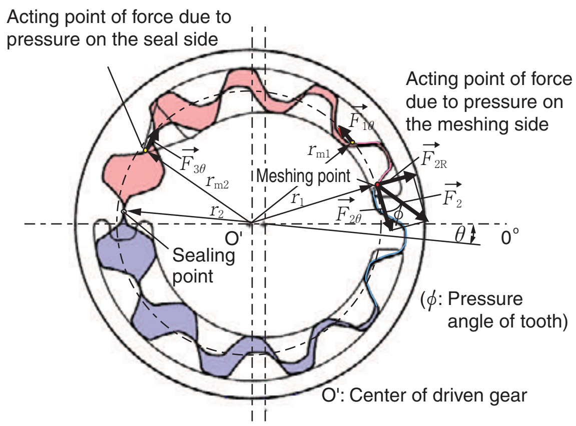

Fig. 6 Forces acting on driven gear

In the same way, only the component of θ direction should be considered for the reaction force $\vec{F}_2$ from the driving gear. Denoting that the pressure angle of the tooth face at the meshing point is ϕ, $|\vec{F}_2|\cos\phi$ means $|\vec{F}_{2\theta}|$.

Disregarding friction and other loss, the moment equilibrium equation around the center of the driven gear O' is expressed by the following equation from these three forces and the radii.

$$|\vec{F}_{1\theta}|\, r_{m1} - |\vec{F}_{3\theta}|\, r_{m2} = |\vec{F}_{2\theta}|\, r_1 \tag{5}$$

The equation (5) can be changed to the following equation to determine $|\vec{F}_{2\theta}|$:

$$|\vec{F}_{2\theta}| = (|\vec{F}_{1\theta}|\, r_{m1} - |\vec{F}_{3\theta}|\, r_{m2})/r_1 \tag{6}$$

By substituting equations (1) through (4) in equation (6):

$$|\vec{F}_{2\theta}| = \Delta pb\,\{(r_1^{\,2}-r_{T1}^{\,2})-(r_2^{\,2}-r_{T1}^{\,2})\}/(2r_1)$$

$$= \Delta pb\,(r_1^{\,2}-r_2^{\,2})/(2r_1) \tag{7}$$

Next, the force and the torque to rotate the driving gear are considered, as shown in **Fig. 7**. Just like the teeth of the driven gear, the forces due to the pressure $\vec{F}_4$ and $\vec{F}_6$ act on the teeth with the meshing point and the sealing point in the driving gear respectively loaded. Also, the reaction force $\vec{F}_5$ acts against the aforementioned force $\vec{F}_2$ on the meshing point of the driven gear and the driving gear. Only the components of force in the θ direction are actually producing the torque to drive the pump. The forces generated by the hydraulic pressure $\vec{F}_4$ and $\vec{F}_6$ are the known values. Also, as in the case of the driven gear, they can replace the equivalent concentrated forces as expressed below:

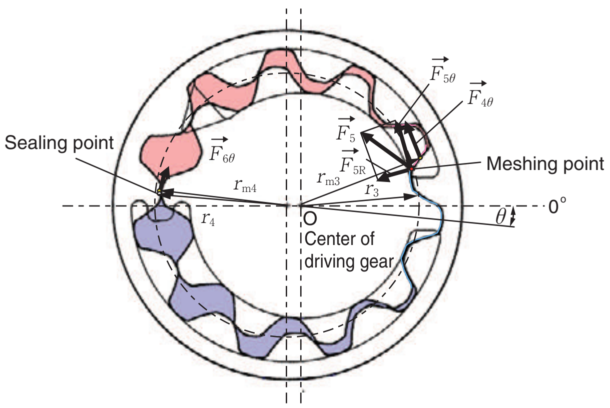

Fig. 7 Forces acting on driving gear

$$|\vec{F}_{4\theta}| = \Delta pb\,(r_{T2}-r_4) \tag{8}$$

$$|\vec{F}_{6\theta}| = \Delta pb\,(r_{T2}-r_6) \tag{9}$$

The radii r_{m3} and r_{m4} from the center of the driving gear O to the points acting $\vec{F}_4$ and $\vec{F}_6$ are expressed as follows.

$$r_{m3} = (r_{T2}+r_4)/2 \tag{10}$$

$$r_{m4} = (r_{T2}+r_6)/2 \tag{11}$$

The force $\vec{F}_5$ acting on the meshing point from the driven gear has the same magnitude as the aforementioned force $\vec{F}_2$, though the direction is opposite. Therefore, the θ direction component $\vec{F}_{5\theta}$ of $\vec{F}_5$ can be expressed as follows:

$$|\vec{F}_{5\theta}| = \Delta pb\,(r_1^{\,2}-r_2^{\,2})/(2r_1)\} \tag{12}$$

Disregarding the friction and other losses, the torque to rotate the driving gear against the moments due to $\vec{F}_{4\theta}$ and $\vec{F}_{5\theta}$ is defined as the theoretical torque T_{th}. This torque can be obtained from the moment around the center of the driving gear O, as expressed by the following equation.

$$T_{th} = |\vec{F}_{5\theta}|\, r_3 + |\vec{F}_{4\theta}|\, r_{m3} - |\vec{F}_{6\theta}|\, r_{m4} \tag{13}$$

Substituting equations (8) through (12) in equation (13),

$$T_{th}(\theta) = \frac{\Delta p}{2} b\{(r_1^{\,2}-r_2^{\,2})\frac{r_3}{r_1}+(r_4^{\,2}-r_3^{\,2})\} \tag{14}$$

In the above equations, the values of r_{T1} and r_{T2} are fixed. Because the coordinates of the meshing point and the sealing point vary according to the rotational angle of the driving gear θ, r_1, r_2, r_3, r_4, r_{m1}, r_{m2}, r_{m3}, r_{m4} as well as $|\vec{F}_{1\theta}|$, $|\vec{F}_{2\theta}|$, $|\vec{F}_{3\theta}|$, $|\vec{F}_{4\theta}|$, $|\vec{F}_{5\theta}|$, $|\vec{F}_{6\theta}|$ are functions of θ. However, they are not expressed as the functions of θ for simplicity of expression.

While equation (14) defines the theoretical torque at each angle, the averaged theoretical torque $T_{th}^{\,*}$ can be calculated by the following equation.

$$T_{th}^{\,*} = \frac{1}{2\pi/z}\int_0^{2\pi/z} T_{th}(\theta)\, d\theta \tag{15}$$

Following the calculation of the theoretical torque, the calculation of the theoretical displacement volume can be performed. Here, the theoretical displacement is related to the theoretical torque as follows. The amount of the work required to drive the pump for one rotation is $T_{th}^{\,*}\times 2\pi$ when the averaged theoretical torque is $T_{th}^{\,*}$. Consequently, because the pressure energy of Δp is given to the oil of volume $V_{th}^{\,*}$, the energy given to the oil is $\Delta p V_{th}^{\,*}$. Disregarding the loss, $2\pi T_{th}^{\,*}$ and $\Delta p V_{th}^{\,*}$ must be equal. Then, the following equation is obtained[5].

$$V_{th}^{\,*} = \frac{2\pi T_{th}^{\,*}}{\Delta p} \tag{16}$$

Equation (16) defines the relationship between the averaged theoretical torque and the theoretical displacement in one revolution. The oil volume delivered from the pump through rotation of $d\theta$ (radians) from the rotational angle θ, $V_{\mathrm{th}}(\theta)$, could be obtained by substituting the averaged theoretical torque $T_{\mathrm{th}}{}^{*}$ in equation (16) for $T_{\mathrm{th}}(\theta)$. Thus, the momentary fluctuation of theoretical displacement would be obtained by equation (17)

$$V_{\mathrm{th}}(\theta)=\frac{T_{\mathrm{th}}(\theta)\,d\theta}{\varDelta p}$$
$$=\frac{b}{2}\{(r_1{}^2-r_2{}^2)\frac{r_3}{r_1}+(r_4{}^2-r_3{}^2)\}d\theta \tag{17}$$

With $d\theta = 2\pi/360$ in equation (17), the fluctuation of the delivered oil per degree can be calculated.

For the internal gear pump with crescent shown in **Fig. 2**, the sealing point always exists on the tip circle radius, and the theoretical torque is expressed as follows:

$$T_{\mathrm{th}}(\theta)=\frac{\varDelta p}{2}b\{(r_1{}^2-r_{\mathrm{T1}}{}^2)\frac{r_3}{r_1}+(r_{\mathrm{T2}}{}^2-r_3{}^2)\} \tag{18}$$

Equation (18) replaces r_2 in the first term and r_4 in the second term of the right side of the equation (14) with r_{T1} and r_{T2} respectively. Since $r_2 > r_{\mathrm{T1}}$ and $r_4 < r_{\mathrm{T2}}$ stand in the pump, the value of the theoretical torque obtained from equation (18) is larger than that obtained from equation (14), and hence the theoretical displacement becomes larger too.

4. 2 Calculation Results

The calculation results based on the assumption of $\varDelta p$= 2 MPa will now be explained. **Table 1** shows the dimensions of the test pump used for the calculation and the experiment. In this internal gear pump, the driving gear rotates for one tooth pitch, while the teeth involved in meshing and sealing in the driving and driven gears change in three patterns. These patterns are shown in **Fig. 8**, where the gears rotate clockwise, and each tooth of both the gears is numbered. Initially, No. 1 driving gear tooth meshes with No. 1 driven gear tooth, while No. 6 driving gear tooth and No. 7 driven gear tooth have the sealing point. When the rotational angle of the driving gear exceeds θ_1, the meshing teeth of both the driving and the driven gears transfer to No. 2, whereas the sealing is still formed by No. 6 driving gear tooth and No. 7 driven gear tooth. When the rotation progress further, the teeth with the sealing point of the driving and driven gears change to the next teeth. **Table 2** summarizes these results.

Table 1 Dimensions of pump

Driving gear	Tip circle radius r_{T2}, mm	34.9
	Root circle radius r_{b2}, mm	27.7
	No. of teeth z	10
Driven gear	Tip circle radius r_{T1}, mm	31.4
	Root circle radius r_{b1}, mm	38.6
	Outer radius r_0, mm	43.2
	No. of teeth z'	11
Gear width b, mm		11.4

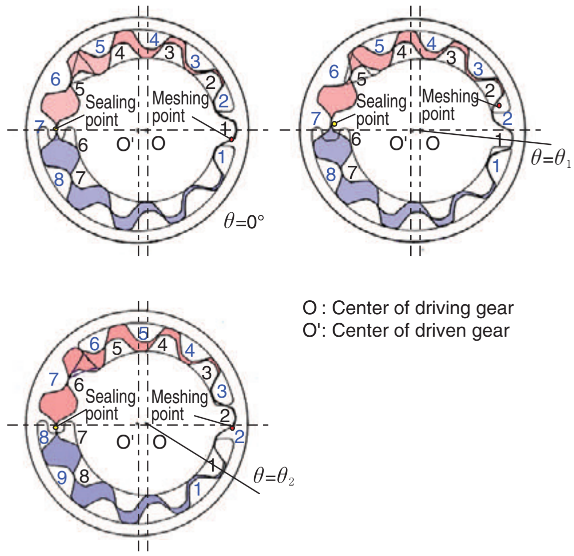

Fig. 8 Transfer of contact points

Table 2 Pattern of number of teeth with contact point

Driving gear rotation angle θ	Meshing tooth No.		Sealing tooth No.	
	Driving	Driven	Driving	Driven
$0\sim\theta_1$	1	1	6	7
$\theta_1\sim\theta_2$	2	2	6	7
$\theta_2\sim 2\pi/z$ (rad)	2	2	7	8

Depending on the transfer of the meshing and sealing teeth, both the forces acting on the teeth and the moment arms change significantly. **Figure 9** shows changes of the magnitudes of the forces $|\vec{F}_{1\theta}|$, $|\vec{F}_{2\theta}|$ and $|\vec{F}_{3\theta}|$ acting on the driven gear teeth. **Figure 10** shows the changes of the magnitudes of the forces $|\vec{F}_{4\theta}|$ and $|\vec{F}_{6\theta}|$ acting on the driving gear teeth. At the rotating angle $\theta=\theta_1$, the forces $|\vec{F}_{1\theta}|$ and $|\vec{F}_{2\theta}|$ as well as $|\vec{F}_{4\theta}|$ ($|\vec{F}_{5\theta}|$ is the same as $|\vec{F}_{2\theta}|$) change suddenly because the meshing point moves from the tooth bottom to the next tooth tip of the driving gear (or from the tooth tip to the tooth bottom of the driven gear) with the transfer of the meshing teeth. The increase and decrease in the forces $|\vec{F}_{1\theta}|$ and $|\vec{F}_{2\theta}|$ of the driven gear are opposite to the $|\vec{F}_{4\theta}|$ of the driving gear.

This is a typical feature of the gear pumps including the external gear pump. On both the driving and driven gears, the forces $|\vec{F}_{3\theta}|$ and $|\vec{F}_{6\theta}|$ acting on the teeth with the sealing point, are relatively small but they cannot be negligible.

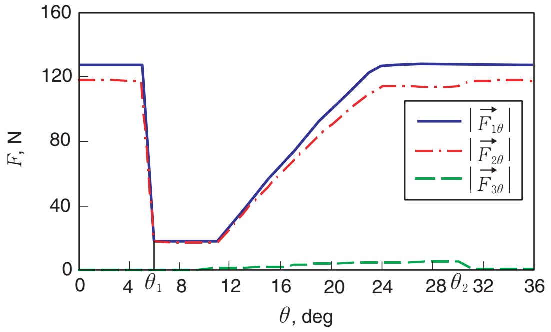

Fig. 9 Change of forces acting on driven gear

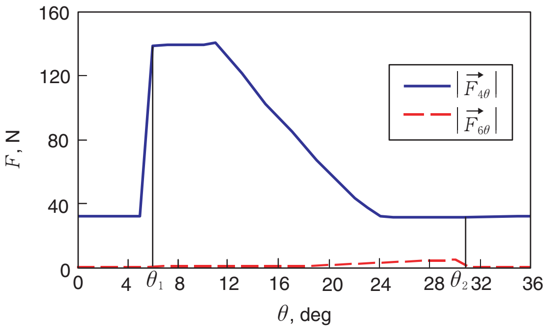

Fig. 10 Change of forces acting on driving gear

The theoretical torque T_{th} to drive the internal gear pump is the sum of the driving torque for the driven gear T_1 (the first term of right side in equation (18)) and the driving gear T_2 (the second term of right side in equation (18)). The calculated results of T_1, T_2 and the theoretical torque T_{th} (T in **Fig. 11**) to drive the pump are shown in **Fig. 11**. Although T_1 and T_2 vary greatly along the rotational angle θ of the driving gear, they cancel out the fluctuations of each other, and the total of them as T_{th} has little fluctuation. Also, from these results, the fluctuation of the theoretical delivery flow rate remains very small, as shown in **Fig. 12**.

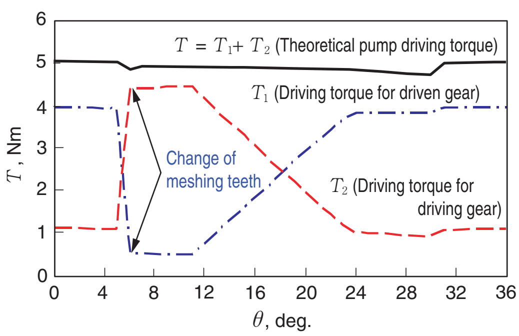

Fig. 11 Torques for driving each gear and pump driving torque

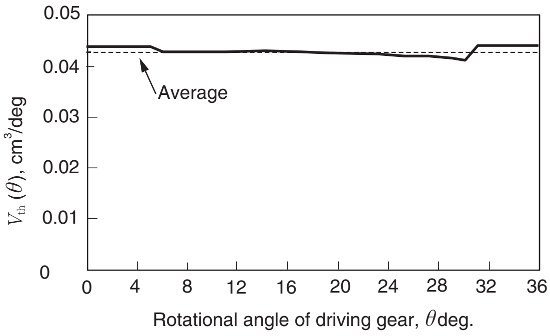

Fig. 12 Change of pump displacement per degree

As the calculated results for the test pump, the values of the averaged theoretical torque T_{th} at Δp= 2 MPa and the theoretical displacement V_{th} were 4.912 Nm and 15.43 cm^3/rev., respectively. Incidentally, the difference between the calculated result by equation (14) and that by equation (18) amounted to about 2%.

5. Experimental Measurement of Displacement

5. 1 Experiment Equipment and Measuring Method

With the experimental apparatus shown in **Fig. 13**, the actual value of the displacement V_{th} for the test pump with the dimensions given in **Table 1** was measured. The simple experimental apparatus consists of the test pump ① driven by a DC motor ② via pulleys and belt. The delivery pressure p_d was measured with a Bourdon's tube pressure gauge equipped at the pump outlet. The delivery flow from the test pump was measured with a flow meter and oil temperature was measured at the pump outlet with a thermistor thermometer. The oil used was commercial mineral oil and its density and viscosity at 80℃ were 810 kg/m^3 and 0.0085 Pa·s, respectively.

The test pump was operated at various pump speeds N, and delivery pressures p_d, and the volumetric flow rate delivered from the test pump Q was measured.

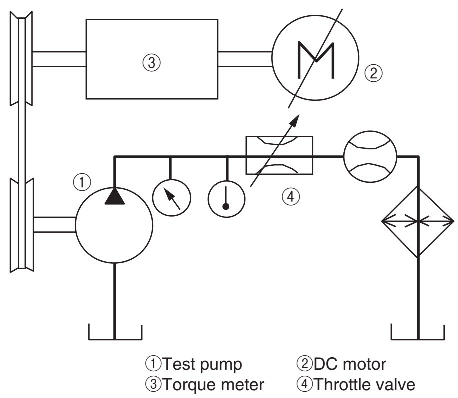

Fig. 13 Test circuit

This time, the theoretical displacement V_{th} was calculated in the following procedure. The volumetric flow rate was measured at various delivery pressures p_d for three pump speeds N. From the results measured at various p_d for each N, the delivery flow rate Q_0 at $p_d = 0$ was then extrapolated. Dividing Q_0 by the pump speed N, the pump displacement V_{th} was obtained.

$$Q_0 = \lim_{p_d \to 0} Q \,(\text{at } N = \text{const.}) \tag{19}$$

$$V_{th} = Q_0 / N \tag{20}$$

5. 2 Experimental Results

The measured delivery flow rates for three pump speeds N with changing delivery pressure p_d are plotted in **Fig. 14**. In **Fig. 15** the values of the pump displacement V_{th} are shown. They were obtained by dividing the delivery flow rate Q_0 at $p_d = 0$ by N for each N.

The experimental result of V_{th} agrees well with the calculated one.

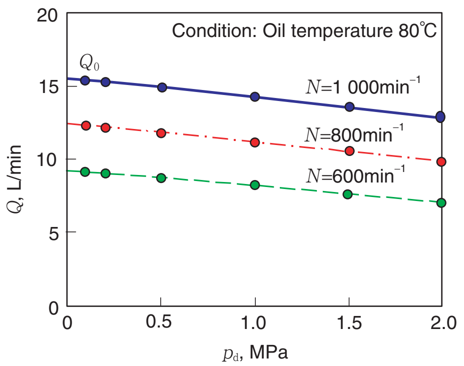

Fig. 14 Real flow of pump

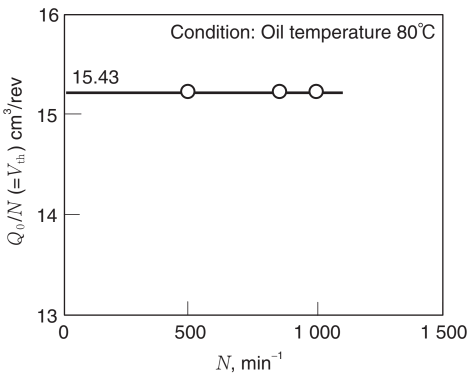

Fig. 15 V_{th} estimated from measured flow

6. Conclusions

In this study, the theoretical torque and the theoretical displacement of the internal gear pump without crescent were investigated. As a result, the following results were obtained:

(1) The calculation method for the theoretical displacement of the pump has been constructed, based on calculation of the theoretical pump driving torque from the forces acting on the gears.

(2) The theoretical displacement value obtained by using this calculation method agrees well with the value obtained by the experiment.

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