Performance Enhancement of Grinding Processes
–Mutual Interaction between the Material Removal Process and the Machine Tool–

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In order to attain the performance enhancement of grinding processes in terms of the high productivity as well as high machining accuracy, it is necessary to establish the reliable physical models of the material removal process and the machine tool performance. In addition to those, it is essential to understand their static and dynamic mutual interactions.

Key Words: grinding process, grinding machine, mutual interaction, stiffness, chatter vibration

1. Introduction: Characteristics of Grinding Processes

The grinding process, a process of material removal using a high-precision grinding wheel with high-hardness, fine abrasive grains, demonstrates its good characteristics to the maximum extent in the processing of hard-to-cut material requiring a high-precision, high-quality surface finish, in which case the chips generated are far smaller than those generated from cutting. Even today grinding remains a critical material removing process, and there are numerous applications regarding which cannot be replaced by any other machining method.

There are several aspects of the material removal process by grinding machines that differ from those by other machine tools. First of all, as shown in Fig. 1, the grinding wheel, used as the machining tool is adjusted in its form as well as in its distribution of abrasives on the working surface. In other words, a grinding machine is required not only to carry out the grinding process but also to execute this adjustment operation (truing and dressing) with high reliability. In the case of a super-abrasive, such as CBN or diamond abrasive grains, the grinding wheel is so hard compared to conventional wheels that it is difficult to adjust the wheel. The quality of this adjustment operation will significantly affect the outcome of the subsequent grinding operation. This necessitates high-precision control of the relative motion between the grinding wheel and truing tool, and particularly, high stiffness of the truing tool system.

Secondly, it must be kept in mind that the behavior of the grinding wheel as an elastic body used as the tool has significant influence on the outcome of the process. Namely, the elastic deformation of the bonding material under grinding resistance will exert significant effect on the material removal process, unlike the case of cutting tools.

In the material removal process, represented by cutting and grinding, the process and the machine tools constitute a closed loop mediated by the forces, heat and deformation, as illustrated in Fig. 2. In order to enhance performance of such processing system, it is essential to understand well the characteristics and interaction of these comprising factors. In this paper we will try to summarize guidelines toward achieving higher performance in the grinding process, shedding light on “the interaction of the process and the machine through force and deformation.”
2. Static Interaction

In order to achieve high processing accuracy and high efficiency, it is essential to provide enhanced static stiffness. The following is an explanation for this requirement of high static stiffness that uses cylindrical plunge grinding as an example. Supposing, as illustrated in Fig. 3, that the feed of the wheel spindle stock is $x$, the reduction of radius of the work is $r$, and the elastic deformation of the machine system generated by the normal component of grinding resistance is $y$, the following relationship stands:

$$ r = x - y $$

(1)

If the machine system has a static stiffness $k$,

$$ y = \frac{F_n}{k} $$

(2)

where, $F_n$ is the normal grinding resistance. Although the above equation (2) provides a simple relationship between the external force (input into machine system) and the deformation (output from machine system), it is important as a model equation providing for static behavior of a machine system. Namely, $1/k$ is referred to as the static compliance that can be considered as the transfer function for the machine system. Assuming that the feed speed of the wheel spindle stock is $v$, and the time is $t$,

$$ x = v_t $$

(3)

Therefore,

$$ r + \frac{F_n}{k} = v_t $$

(4)

Here, assuming that the grinding power is proportional to the material removal per unit time ($Q = b \pi d_e \frac{dr}{dt}$), or the grinding rate, and supposing that $F_t$ is the tangential grinding resistance, $b$ is the grinding width, $d_e$ is the work diameter, $V$ is the wheel peripheral velocity, and $\gamma$ is the proportional constant,

$$ F_t = \gamma b \pi d_e \frac{dr}{dt} $$

(5)

Thus, the following equation that provides the static normal grinding resistance is derived:

$$ F_n = \gamma' b \pi d_e \frac{1}{V} \frac{dr}{dt} $$

(6)

where, $\gamma'$ represents the ratio of the tangential grinding resistance to the normal grinding resistance, $\gamma' = F'_n/F_t$.

The equation (6) is deemed as a model equation of grinding process. From the above relationship, the following differential equation is obtained.

$$ r + \frac{\gamma' b \pi d_e}{k} \frac{1}{V} \frac{dr}{dt} = v_t $$

(7)

Since the cylindrical plunge grinding is found to be approximated to a first order time lag system, solution of the equation (7) will be,

$$ r(t) = v_t - v_t \left( 1 - e^{-\frac{t}{\tau}} \right) $$

(8)

where, $\tau$ represents a time constant of the first order time delay system which is given as follows:

$$ \tau = \frac{\gamma' b \pi d_e}{kV} $$

(9)

The equation (9) is graphically represented in Fig. 4 for grinding time using time constant as the parameter, provided that in this diagram, the grinding cycle is supposed to be comprised of a constant in-feed rate and the spark-out grinding after stopping the infeed motion. The solid line in the diagram represents the movement of the wheel spindle stock, whereas the dotted lines indicate the actual reduction of workpiece radius. Inclination of the dotted lines indicates the rate of reduction of workpiece radius, which is noted to be inconsistent with the wheel feed rate except for the steady-state area. The rate of radius reduction increases gradually from the start of grinding until it becomes parallel with the wheel feed rate when it enters into steady-state grinding. When the wheel ceases to be fed and the spark-out grinding starts, the rate of radius reduction phases down to zero. Such time delay is promoted by the increase in the time constant, namely reduction of stiffness of the machine system (equation (9)). On the other hand, the time constant can be reduced by increasing the wheel peripheral speed, making the most of high-speed grinding.

![Fig. 3 Elastic deformation model of grinding machine](image_url)
In the grinding process, elastic deformation of the grinding wheel cannot be ignored. Since the wheel is deformed at the contact with the workpiece due to grinding resistance, contact stiffness must be taken into account in equation ArgumentException. Figure 6 shows the results of measuring contact stiffness at the working surface of the grinding wheel, which indicate a nonlinear characteristic of higher stiffness with the increased deformation.

During actual grinding operations, the grinding wheel becomes dull as time passes, causing the constant \( c \) to change. This gives rise to a change of the time constant, which affects the result of grinding. As mentioned earlier, it is desirable in grinding that the time constant be as small as possible. To this end, it is necessary to keep the static stiffness of the machine system at a high level. As revealed in equation (9), the time constant of grinding is determined through interaction between the process and the machine stiffness.

Figure 4 shows that the lower the stiffness of the machine system is, the longer the time required for spark-out grinding will be. Also, if it is supposed that grinding is finished after a certain period of time without spark-out grinding, the greater the time constant is, i.e. the lower the machine system stiffness is, the greater the dimensional errors are, or to be more specific, the greater the residual depth of cut is. Furthermore, variation of work radius reduction rate at the end of grinding may give rise to variation in roughness of the finished surface.

Stiffness of a machine system is determined by elastic deformation of each element comprising the grinding machine as well as deformation and slackness at various links including the slideway surface and bearings, as illustrated in the model diagram in Figure 5. Here, the crux of the problem lies in the elements that exist on the route through which the grinding resistance is transmitted. Supposing that overall stiffness \( k_r \) of a grinding machine approximates series linkage of the stiffness of each link, the following equation stands:

\[
\frac{1}{k_r} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n}
\]  

Equation (10) suggests two significant points. One is that in order to enhance overall stiffness, it is effective to make the route of grinding resistance transmission (or flow of force) as short as possible. The other point is that good balance should be kept between the stiffness of each composing element and link, doing away with even one link having extremely low stiffness. This is because, as evident from equation (10), the overall stiffness is affected most significantly by a link of low stiffness.

3. Dynamic Interaction

In order that high precision and high efficiency be maintained in the grinding process, no vibration or chattering should be allowed during the process. Degradation of workpiece contour and roughness due to vibration in grinding can be more severe than that caused by vibration in machining.

Vibration generated in grinding is classified into two kinds by cause: forced vibration, and self-excited vibration (Figure 7). Forced vibration is that which is generated when...
there exists a vibration source that forcibly drives the mechanical structure to vibrate. In the case of a grinding machine, imbalance of the grinding wheel represents the most serious source of vibration. With the exception of internal grinders, grinding wheels have such great mass and are driven at such high speeds that the vibration exciting force due to imbalance is markedly greater than that in other machine tools. Therefore it is most important to ensure high dynamic stiffness of the wheel spindle system and thereby to minimize the amplitude of the forced vibration. While the amplitude \( y \) of the vibration generated by the forced exciting force \( F \) is a function of the vibration frequency, it becomes largest when the frequency coincides with the natural frequency of the machine system, i.e. at resonance. The value is expressed as follows:

\[
\frac{y}{F} = \frac{1}{2\kappa \zeta}
\]

(1)

where, \( \zeta \) represents a damping ratio. To increase dynamic stiffness, it is important to improve not merely the static stiffness \( k \), but also the damping ability.

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**Fig. 7** Vibration phenomena in the grinding process (1)

The vibration source No. 2 is the self-exciting vibration due to the regenerative effect. Although we will not discuss here the details of its theoretical background, it is an unstable vibration based on a time delay phenomenon. Supposing that a relative displacement is generated between the grinding wheel and the workpiece for one reason or another, its effect will be left on the work surface in the form of cyclic waviness. When the same affected area on the work comes back to grinding after one revolution, its effect shows up as a change in cut depth. This unstable vibration generated by vibration that took place one revolution earlier is called self-excited vibration caused by the regenerative effect. As a matter of course, this problem can also happen in cutting. In the case of grinding, however, the grinding wheel may also be susceptible to cyclic waviness generated through vibration and wear. In this way, the vibration phenomena can be exceptionally involved due to existence of the regenerative effect on the grinding wheel surface (Fig. 8).

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**Fig. 8** Vibration phenomena in the grinding process (2)

The phenomenon of reproduction type self-excited vibration in grinding are summarized as follows:

1. Self-excited vibration due to the regenerative effect on the workpiece periphery can progress so fast that it becomes impossible to perform the process under such conditions. Self-excited vibration of this type is liable to be generated when the peripheral speed of the workpiece is high.

2. On the other hand, by reducing the workpiece peripheral speed, self-excited vibration due to the regenerative effect on the workpiece can be restrained, although it then helps generate self-excited vibration due to the regenerative effect on the grinding wheel working surface. Although vibration of this type progresses relatively slowly, it has been theoretically demonstrated that most of the grinding conditions fall within an unstable area. Namely, because the wheel surface must be adjusted (trued or dressed) at a point when the gradually progressing amplitude of vibration reaches the allowable limit, the criterion of service life of the grinding wheel can be reached. It is, therefore, necessary to select such grinding conditions that would restrain the progress of vibration and thereby extend the service life of the grinding wheel.

The reason why these two regenerative effects are subject to the influence of the peripheral speed can be considered to lie in the geometrical interference action between the grinding wheel and the workpiece as illustrated in Fig. 9. Namely, if the work peripheral speed is reduced gradually with the frequency and the amplitude kept constant, the amplitude of the waviness on the workpiece surface is expected to be reduced due to relative vibration through geometrical interference effect, and hence the regenerative effect is expected to taper off. Such interference effect must also exist on the periphery of the grinding wheel, though it can be neglected under normal operating conditions because its peripheral speed is much higher than that of the workpiece.

Concerning stability of self-excited vibration due to the regenerative effect, an outline discussion based on the vector locus diagram of dynamic compliance of the grinding machine is shown in Fig. 10.
the distance from the origin to the curve denotes the amplitude response of the machine system against the vibration exciting factor, and the angle to the horizontal axis represents the phase difference. In other words, by the use of the vector locus diagram, vibration characteristics of a machine system can be perfectly described in one curve. The horizontal component of this curve represents the real part of the response characteristics, and the vertical component indicates the imaginary part. Also, since the phase difference is 90, the intercept on the vertical axis indicates the response at resonance.

Fig. 9 Geometric interference between the grinding wheel and the work piece

Fig. 10 Vector locus of dynamic compliance

Although we are skipping detailed theoretical explanations, the stability of self-excited vibration due to the regenerative effect can be expressed as the magnitude of the maximum negative real part of the vector locus, or extent to which it sticks out toward the negative side on the horizontal axis. The smaller the leftward stick-out for a grinding machine is, the better dynamic stability the machine can be said to have. Generally, the dynamic compliance of a machine system, or the dynamic transfer function, i.e. the vector locus, is expressed in the following equation:

\[ \frac{\text{Gain}}{\text{Phase}} = \frac{V}{J } \left[ \text{Re}G(j\omega) + \text{Im}G(j\omega) \right] \]  \hspace{1cm} (12)

where,

\[ u = \cos \alpha \cdot \cos (\alpha - \beta) \]  \hspace{1cm} (13)

where, \( u \) is a factor called directional factor, \( \alpha \) is the direction of the natural vibration mode of a structure relative to the normal direction of ground surface, and \( \beta \) stands for the direction of vibration exciting force relative to the normal direction of ground surface. In other words, it denotes a factor to derive the component in a direction normal to the ground surface of the natural vibration mode of the structure generated by the vibration exciting force acting in \( \beta \) direction. As revealed in the equation (12), the smaller the directional factor is, the smaller the magnitude of the maximum negative real part in the dynamic characteristics of the machine system will be, hence increasing stability against the self-exciting vibration. This effect is almost as great as that of static stiffness. In designing a grinding machine with high stability, it is necessary to give sufficient consideration to the direction of the natural vibration mode (Fig. 11). In discussion of the dynamic interactions, it is essential to be aware of the model equation (12) of the machine structural system.

In the discussion of grinding process stability, a model equation concerning dynamic grinding resistance is required. Specifically, it is necessary to know the response of the grinding process in conjunction with material removal while relatively vibrating in the direction of cutting depth (with the input of relative vibration amplitude between the grinding wheel and the workpiece, response in the form of output of varying grinding resistance). Although we will not go into details here, the results of theoretical analyses suggest that there exists a term which is proportional to the speed of relative vibration.\(^2\) The reason for this is that the grinding wheel and the workpiece are in an area of contact. As the force proportional to the vibration speed denotes a damping force, a stability enhancing factor is intrinsic in the grinding process.

In designing dynamic grinding processes, the elastic deformation characteristics, i.e. the effect of contact stiffness at contact with the workpiece, is a critical factor. While a low grade grinding wheel with low contact stiffness provides high stability, such a grinding
wheel is normally more liable to generation of wear and development of self-excited vibration due to the reproduction effect on the wheel working surface. In this way, the characteristics of grinding wheel used as a cutting tool can exert complex effects on grinding process stability.

4. Measures to Improve Dynamic Stability

The generation of self-excited vibration does not merely degrade machining accuracy, but also leads to the decline of process efficiency. Therefore it is critical to restrain such vibration for achievement of high performance grinding. Vibration restraining measures can be achieved only by considering the behaviors of the process and the grinding machine as well as the interaction thereof.

Figure 12 shows this concept. The straight line drawn parallel to the imaginary axis in the negative real section represents the grinding process, which moves close to the imaginary axis when, for instance, $\gamma$ or $b$ increases in the equation (5). The vector locus, as suggested in the above section, indicates vibration characteristics of the grinding machine. The result of stability analysis has demonstrated that the process system becomes unstable with generation of self-exciting vibration when the straight line has intersecting points with the vector locus. Therefore, vibration restraining measures should be such that these intersections do not happen. Specifically, the following countermeasures are derived:

(a) Change the grinding conditions in such a way that moves the line parallel to the imaginary axis leftward.

(b) Increase the static stiffness of the grinding machine $k$, or reduce directional factor $u$ so as to minimize the maximum standoff of the vector locus in the negative real section.

(c) Increase the damping ratio $\zeta$ of the grinding machine.

(d) Move the entire vector locus rightward, which can practically be achieved to a certain extent by the use of a lower bonding grinding wheel that has lower contact stiffness.

(e) Disturb the reproduction effects by staggering the phases of reproduction effects on the workpiece and the grinding wheel surfaces. This can be achieved by changing their rotational speeds.

Starting with an understanding of the material removing process and the behavior of machine tools, and through considering the interaction thereof, it is possible to work out logical approaches to restraining vibration.

5. Conclusion

With a view to gaining guidelines for achieving high-precision, high-efficiency grinding, we have pointed out that it is essential to grasp the behavior of the process and the machine as well as to understand the interaction thereof. As far as the process is concerned, first of all it is necessary to search for a physical model that explains how grinding resistance changes depending on the process condition, and then to search for deformation response against static and dynamic external forces on the grinding machine.

References


Fig. 12 Strategies for suppressing the regenerative chatter vibration