Mathematical Model for Characteristics of Oil Pump Driven by Electric Motor

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This report presents a mathematical model for characteristics of a motor-driven oil pump in which an electric motor and hydraulic pump have been integrated into a single unit. Characteristics of the motor-driven oil pump should be predicted at the design stage. However, calculation thereof is not simple because the characteristics of the electric motor and oil pump influence one another. Therefore, a mathematical model to predict the characteristics of the assembled electric motor-driven oil pump has been established based on investigation of individual pump and motor characteristics. Also, actual characteristics of the electric motor-driven oil pump were investigated and compared with the values calculated from the model.

Key Words: *hydraulic power system, electric motor-driven oil pump, mathematical model, pump characteristic, motor characteristic, system efficiency*

1. Introduction

Electric motor-driven oil pumps have already been developed and commercialized for use with automotive hydraulic power steering systems for cases in which there is no space around the engine to install the conventional engine-driven oil pump and/or in order to achieve significant energy-savings[1), 2)]. In the case of electric motor-driven oil pumps used with hydraulic power steering systems, changing the flow rate from the oil pump by controlling the speed of the electric motor according to driving situations, i.e. decreasing the flow rate delivered from the oil pump during non-steering operation, can improve fuel economy[1)-3)]. Electric motor-driven oil pumps are also used in continuously variable transmissions (CVT) for hybrid vehicles[4), 5)], which cannot have engine-driven accessories. In addition, a system to stop the engine during idling has been developed for use in vehicles equipped with a CVT or an automatic transmission (AT) in order to save energy[6)-8)]. In such a system, an electric motor-driven pump instead of an engine-driven pump delivers oil during idling and maintains an adequate oil pressure to engage clutches for smooth starting. Furthermore, a dual clutch transmission (DCT), a new transmission in which the oil to operate the clutches is stored by an intermittent operation of an electric motor-driven oil pump, has recently been developed[9)].

In designing an electric motor-driven oil pump, predicting the system's overall characteristics is very important. Especially in a system utilizing a 12 volt battery, because the pump operation under high pressure conditions increases highly the electric current and influences heat generation and element reliability in the system, predicting the electric current in the design stage is important.

This report describes the construction of a mathematical model expressing the characteristics and system efficiency of an electric motor-driven oil pump system based on individual models presenting actual characteristics of the oil pump and electric motor. Comparisons between the values calculated from this mathematical model and the actually measured ones are shown.

2. Nomenclature

E_m : Motor input voltage (V)
I_m : Motor current (A)
N : Motor speed (min^{-1})
p_d : Pump delivery pressure (MPa)
p_s : Pump suction pressure (MPa)
Δp : Pressure difference (= $p_d - p_s$) (MPa)
Q : Flow rate (cm^3/s)
T : Pump driving torque (Nm)
V_{th} : Theoretical pump displacement per revolution (cm^3/rev.)
η_{pv} : Volumetric efficiency of pump
η_{pm} : Mechanical efficiency of pump
η_{pt} : Total efficiency of pump (= $\eta_{pt} \times \eta_{pm}$)
η_{mt} : Total efficiency of motor
η : System (pump and motor) efficiency (= $\eta_{pt} \times \eta_{mt}$)

3. Structure and Features of Electric Motor-Driven Oil Pump

Figure 1 shows a cross-sectional view of an electric motor-driven oil pump. Here, a balanced vane pump is used as an example. The concept and analytical method developed in this report can be also applied to different pump types because flow rate and driving torque characteristics of the balanced vane pump and other pumps are fundamentally similar. A balanced vane pump is composed of a cam ring with an elliptic inner bore, a rotor which houses a series of radially-movable vanes, and two side plates, one on each side of the rotor. The balanced vane pump has a double displacement process with two suction/discharge cycles per revolution and is designed so that both the pump's suction and delivery ports are diametrically opposed to provide complete balance of all internal radial forces. Because no bending force acts on the pump shaft, only torsional strength needs to be considered in designing. With balancing radial forces, the bearing or bushing can be eliminated in the pump, contributing to compactness.

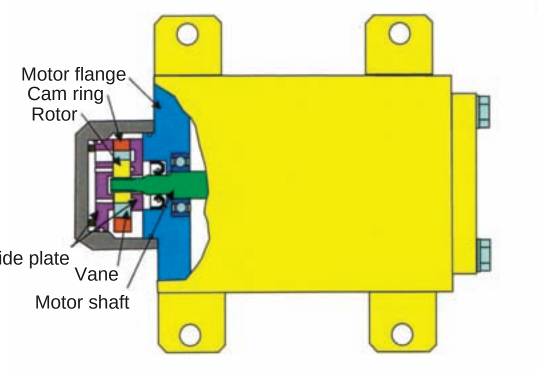

Fig. 1 Structure of electric motor-driven oil pump

The electric motor-driven oil pump shown in **Fig. 1** does not possess its own shaft. The pump rotor is connected to the end of the shaft of the electric motor through a very loose-fitting spline and directly driven by this shaft. The motor flange functions as a front housing of the pump. The rotor and 10 vanes rotate between the two side plates with a very slight running clearance. During pump operation, the vanes are always pushed from the bottom by the delivery pressure and rotate with the loads imposed by the vane tip on the cam contour. The pump is constructed so that the delivery pressure pushes one of the side plates, and the clearance between the rotor and side plates decreases to prevent an increase of leakage flow and to maintain a high volumetric efficiency on high-pressure condition. Specifications of the oil pump (vane pump) are given in **Table 1**.

The electric motor is a DC motor with brushes and ferrite magnets that operates with a 12 volt battery. Its specifications are given in **Table 2.**

Table 1 Specifications of oil pump

Theoretical displacement, cm^3/rev.	1.50
Rotor diameter, mm	ϕ23.6
Width of cam ring, mm	5.3
Thickness of vane, mm	0.9
No. of vanes	10
Relief pressure, MPa	10

Table 2 Specifications of electric motor

Size (diameter × total length), mm	ϕ110 × 148
No. of brushes	4
Motor input voltage, V	13.5
Maximum motor speed, min^{-1}	3 400

4. Construction of Mathematical Model of Characteristics in Electric Motor-Driven Oil Pump

4. 1 Mathematical Model of Oil Pump Characteristics

First, the relationship between delivery pressure and driving torque, the fundamental characteristic of the oil pump is considered. For oil pump torque characteristics, Wilson's mathematical model is well known[10]. For the vane pump, a modified model based on Wilson's model was also proposed[11]. However, Wilson's model can usually be applied to various oil pumps including the vane pump as long as changes of pump speed and oil temperature are not extensive. Actually, the oil pump driving torque increases linearly with an increase in the pressure, and the model indicates this fact.

Figure 2 shows the relationship between the pump delivery pressure p_d and the driving torque T. In this T, the friction torque at the pump shaft due to the friction of bearing and oil seal is eliminated. As seen from this figure, T increases linearly from T_0 of Y-intercept (T at p_d=0) with a slope α for an increase of p_d. Although the horizontal axis should be indicated with Δp (=p_d–p_s), using Δp is not practical. Supposing p_s is zero for the purpose of simplification, T can be expressed as the

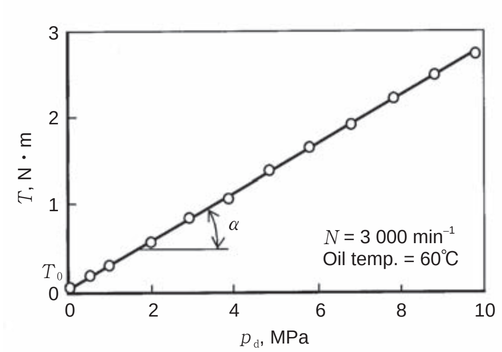

Fig. 2 Relationship between oil pump pressure and torque

following equation:

$$T = T_0 + \alpha p_d \tag{1}$$

For the flow characteristics, it is important to understand the leakage flows in the oil pump. **Figure 3** shows the relationship between the delivery pressure p_d and the leakage flow Q_L in the oil pump used in this study. As seen from **Fig. 3**, Q_L increases linearly with an increase in p_d. This fact suggests that the leakage flows in the oil pump, which are directly proportional to pressure, are actually flows between parallel plates or parallel disks[12]. In the case of an internal gear pump such as a gerotor pump without a crescent to prevent the leakage flow between tips of driving and driven gears, the leakage flow in this area would change as a quadratic function of the pressure because of the flow in an orifice. Although a part of the leakage flow depends on the pump speed N, its influence is relatively small. Then, the effect of the pump speed on the leakage flow can be considered to be negligible.

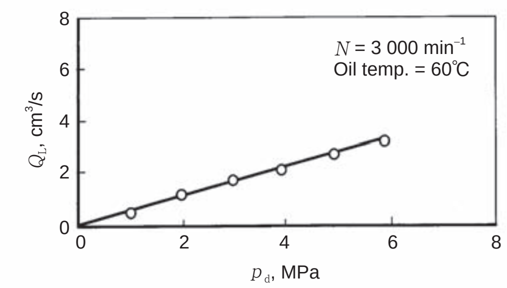

Fig. 3 Characteristic of leakage flow in oil pump

Figure 4 shows the relationship between pump delivery pressure p_d and flow rate Q. With an increase in p_d, each Q decreases linearly from Q_0 of Y-intercept (Q at p_d=0) with a slope of β or β'. The slopes β or β' are almost identical, and Q is expressed by the following equation:

$$Q = Q_0 - \beta p_d \tag{2}$$

where Q_0 is the ideal flow rate for each N and can be defined as the following equation:

$$Q_0 = V_{th} N/60 \tag{3}$$

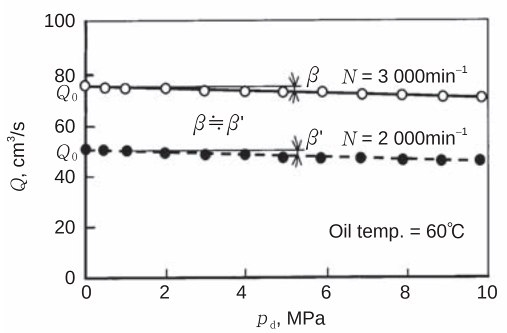

Fig. 4 Relationship between oil pump pressure and flow rate

Pump performance is usually presented as three efficiencies: the volumetric efficiency η_{pv}, the mechanical efficiency η_{pm}, and the total efficiency η_{pt}. They are defined by the following equations, respectively:

$$\eta_{pv} = \frac{Q}{Q_0} = \frac{Q}{V_{th} N/60} \tag{4}$$

$$\eta_{pm} = \frac{T_{th}}{T} = \frac{V_{th}}{2\pi} \frac{p_d}{T} \tag{5}$$

$$\eta_{pt} = \eta_{pv} \times \eta_{pm} = \frac{p_d Q}{T\omega} \tag{6}$$

Figure 5 shows the efficiencies of the oil pump used in this study.

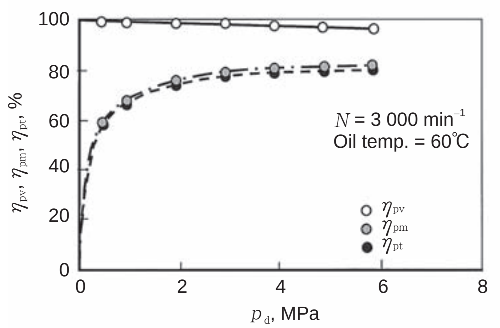

Fig. 5 Efficiencies of oil pump

4. 2 Mathematical Model of Electric Motor Characteristics

Secondly, the characteristics of the electric motor driving the oil pump are considered. **Figure 6** shows the characteristics of the motor speed N against torque T. With an increase in T, N decreases, deriving from the drop of the net input voltage of the motor due to an increase of the electric current. The dotted line is the line connecting the measured values, while the solid line is the approximation of N with a straight line. Although the actual change of N has a nonlinear part, the relationship between N and T could be expressed by the following equation for simplification:

$$N = N_0 - \gamma T \tag{7}$$

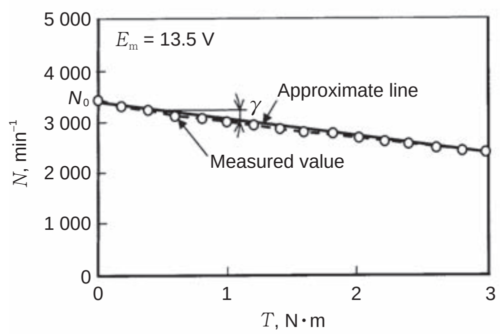

Fig. 6 Relationship between electric motor torque and motor speed

The motor speed at T=0 of Y-intercept N_0 depends on the motor input voltage E_m. **Figure 7** shows the relationship between N_0 and E_m. With an increase in E_m, N_0 increases linearly, and the line has an X-intercept of E_0. The slope of the line is determined by the proper value of the electric motor (the torque constant of the electric motor K_e). By using K_e, N_0 is expressed by the following equation:

$$N_0 = \frac{60}{2\pi}\frac{E_m - E_0}{K_e} \quad (8)$$

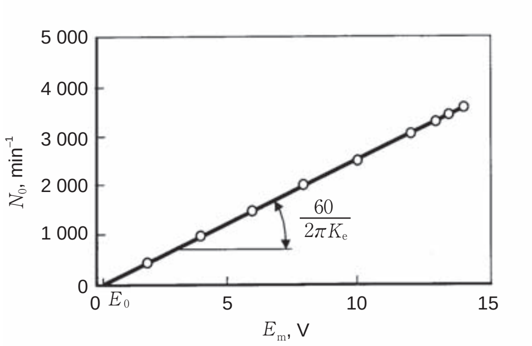

Fig. 7 Relationship between electric motor input voltage and motor speed

In addition, the motor current I_m increases linearly from Y-intercept I_0 with an increase in T. The slope of the line is concerned with K_e, and I_m is expressed by the following equation:

$$I_m = I_0 + \frac{T}{K_e} \quad (9)$$

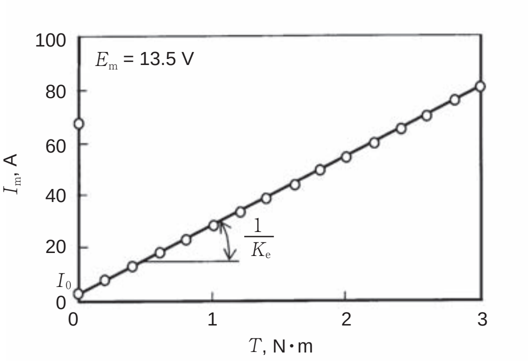

Fig. 8 Relationship between electric motor torque and current

The properties of the electric motor used in this study, i.e. E_0, γ, K_e in **Figs. 7** and **8**, and I_0, are shown in **Table 3**.

Table 3 Properties of electric motor

E_0, V	0.25
γ, (min^{-1})/(N・m)	397.5
K_e, N・m/A	0.0372
I_0, A	3.2

4. 3 Mathematical Model of Electric Motor-Driven Oil Pump Characteristics

From the individual mathematical models of the oil pump and the electric motor explained above, the mathematical model for the characteristics of the electric motor-driven oil pump assembly is obtained.

Substituting equations (1) and (8) for (7), N is expressed as the following equation:

$$N = \frac{60}{2\pi}\frac{E_m - E_0}{K_e} - \gamma\,(T_0 + \alpha p_d)$$

$$= C_N - \alpha\gamma p_d \quad (10)$$

where $C_N = 60\,(E_m - E_0)/(2\pi K_e) - \gamma T_0$

Substituting equations (3) and (10) for (2), Q is expressed by the following equation:

$$Q = \frac{V_{th}}{60}\left\{\frac{60\,(E_m - E_0)}{2\pi K_e} - \gamma T_0 - \alpha\gamma p_d\right\} - \beta p_d$$

$$= C_q - \left(\frac{V_{th}}{60}\alpha\gamma + \beta\right) p_d \quad (11)$$

where $C_q = V_{th}\left(\frac{E_m - E_0}{2\pi K_e} - \frac{\gamma T_0}{60}\right)$

Further, by substituting equation (1) for (9), the following equation can be obtained.

$$I_m = I_0 + (T_0 + \alpha p_d)/K_e$$

$$= C_i + \frac{\alpha}{K_e} p_d \quad (12)$$

where $C_i = I_0 + T_0/K_e$

C_N, C_q and C_i in equations (10) to (12), which are obtained from the characteristics of the actual oil pump and electric motor, are shown in **Table 4**.

Table 4 Properties of electric motor-driven oil pump

C_N, min^{-1}	3 311
C_q, cm^3/s	82.8
C_i, A	4.52

By using the equations of this mathematical model, the total efficiency of the pump η_{pt}, the total efficiency of the motor η_{mt}, and the system efficiency η are expressed as follows:

$$\eta_{pt} = \frac{p_d Q}{\omega T} = \frac{60 p_d Q}{2\pi N T}$$

$$= \frac{60 p_d \{C_q - (V_{th}\alpha\gamma/60 + \beta)\,p_d\}}{2\pi\,(C_N - \alpha\gamma p_d)\,(T_0 + \alpha p_d)} \quad (13)$$

$$\eta_{mt} = \frac{\omega T}{E_m I_m} = \frac{2\pi}{60}\frac{NT}{E_m I_m} = \frac{2\pi}{60}\frac{(C_N - \alpha\gamma p_d)(T_0 + \alpha p_d)}{E_m\left(C_i + \frac{\alpha}{K_e} p_d\right)} \qquad (14)$$

$$\eta = \frac{p_d Q}{E_m I_m} = \frac{p_d\left\{C_q - \left(\frac{V_{th}}{60}\alpha\gamma + \beta\right)p_d\right\}}{E_m\left(C_i + \frac{\alpha}{K_e} p_d\right)} \qquad (15)$$

5. Experimental Results of Electric Motor-Driven Pump

5. 1 Comparisons of Characteristics between Calculation and Experiment

Figure 9 shows the characteristics of the electric motor-driven oil pump consisting of the oil pump having the characteristics shown in **Figs. 2** to **5** and the electric motor having the characteristics shown in **Figs. 6** to **8**. Changes of the flow rate from the oil pump Q, the speed of the electric motor (oil pump) N, and the electric current I_m against the oil pump delivery pressure p_d in this figure are measured at E_m=13.5 V and an oil temperature of 60℃. The solid, dot-dash, and dotted lines indicate Q, N and I_m calculated by using equations (10), (11) and (12), respectively. These values calculated from the mathematical model agree well with the experimental ones, although the characteristics of the oil pump and the electric motor are linearly approximated. This model enables us to predict the characteristics of the electric motor-driven pump on various conditions.

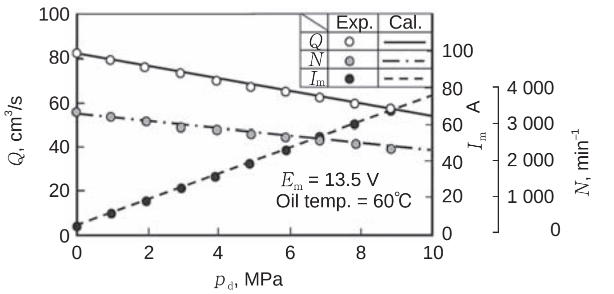

Fig. 9 Flow rate, electric current and motor speed in relation to delivery pressure

Figure 10 shows various efficiencies of the electric motor-driven oil pump. As shown in **Fig. 9**, the solid, dot-dash, and dotted lines indicate η_{pt} (the total efficiency of the pump), η_{mt} (the total efficiency of the motor), and η (the system efficiency of the pump and motor), respectively. As in the case of **Fig. 9**, the predicted values calculated from the mathematical model agree well with the experimental results.

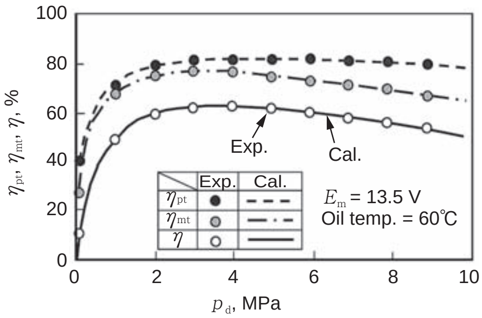

Fig. 10 Efficiencies of electric motor-driven oil pump

5. 2 Pressure Pulsation and Noise Characteristics

In a hydraulic system using an electric motor-driven oil pump installed in a vehicle, it is desired that the pressure pulsation of oil delivered from the oil pump and the operating noise are low. Accordingly, the pressure pulsation and noise of the electric motor-driven oil pump were experimentally investigated.

Figure 11 shows the measured results of the pressure pulsation at various delivery pressures p_d on the condition of E_m=13.5 V and an oil temperature of 60℃. The figure shows the relationship between the frequency and the pressure pulsation levels analyzed using Fast Fourier Transformer (FFT). Because the motor speed decreases gradually with an increase in p_d, the frequency of the peak becomes lower. The pressure pulsation level at the 1st harmonic order of the vane pitch (10th of the pump revolution) for each p_d is the largest. Although the pressure pulsation level rises gradually, it stays below 0.015 MPa even at p_d=9 MPa.

Figure 12 shows the relationship between the pump delivery pressure p_d and the noise level in the electric motor-driven oil pump assembly at the motor input voltage E_m of 13.5 volts. The noise is measured by using A-frequency-weighting with the microphone set at a distance of 100mm from the electric motor. The sound level is 70 dB(A) at a low p_d and rises with an increase in p_d. However, it is below 80 dB(A) even if p_d rises up to about 10 MPa.

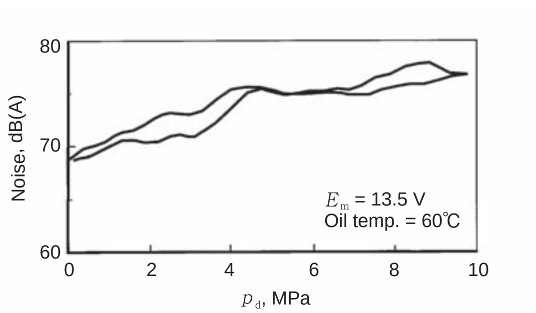

Fig. 12 Noise characteristic of electric motor-driven oil pump

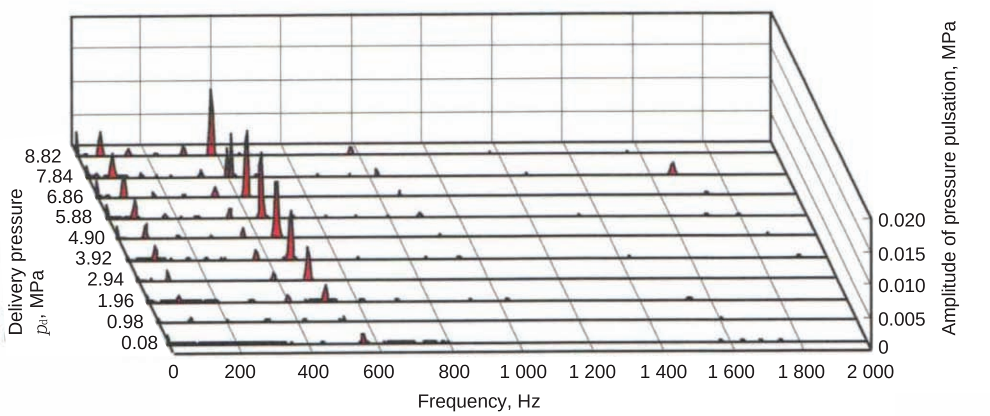

Fig. 11 Pressure pulsations of electric motor-driven oil pump

6. Conclusions

A mathematical model for the characteristics and the efficiencies of electric motor-driven oil pump systems was established based on individual models presenting the actual characteristics of the oil pump and the electric motor. It is verified that the values of the efficiencies as well as the flow rate and the electric current against the delivery pressure calculated by using the mathematical model agree well with experimental ones. With this mathematical model, the characteristics of electric motor-driven pumps under various motor input voltage and delivery pressure conditions can be predicted accurately. Furthermore, it is verified that the actual pressure pulsations and noise of the electric motor-driven oil pump are sufficiently low.

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