Theoretical Analysis of Mechanical Efficiency in Vane Pump

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This paper describes the theoretical analysis on the mechanical efficiency of a hydraulic balanced vane pump and presents a design concept to decide values such as vane thickness and cam lift to improve its mechanical efficiency. The friction torque characteristics of the balanced vane pump, especially the friction torque caused by the friction between a cam contour and vane tips are both experimentally and theoretically investigated. Although the friction force increases with vane thickness increase due to increase in the vane force, the mechanical efficiency of the vane pump does not decrease when the cam lift becomes large as the vane thickness is increased. An important parameter ε defined as the ratio to be obtained by dividing the cam lift by the vane thickness determines essentially the mechanical efficiency of the vane pump. The pumps with the same ε have the same mechanical efficiencies, even if the cam lift or the vane thickness varies. A larger value of ε leads the vane pump to obtain a higher mechanical efficiency. Additionally reducing the friction coefficient at the vane tip contributes to an improvement of the mechanical efficiency.

Key Words: *hydraulic power system, balanced vane pump, friction torque, mechanical efficiency, cam lift, vane thickness*

1. Introduction

A balanced vane pump, now widely used in many hydraulic power systems because of its compactness, lightweight and low cost, is especially suitable for a hydraulic power source in a power steering system of a vehicle, in which low pressure pulsation and low noise are required. When designing a hydraulic pump including the vane pump, the mechanical efficiency as well as the volumetric efficiency constitutes a key factor in evaluating the pump performance.

Friction torque characteristics of various pumps and motors have already been studied and mathematical models of the friction torque have been proposed[1)-4)]. Authors have also proposed a new mathematical model of the friction torque characteristics for the vane pump because the previously proposed models were not suitable for the vane pump[5)]. These mathematical models were constructed conceptually or only to fit measured results in the actual pump torque, but not theoretically. And the relation between the dimensions of the pump parts and its friction torque in the vane pump has yet to be clarified, and it is generally held that the friction torque in the vane pump has been insufficiently analysed.

Therefore, the friction torque characteristics, particularly the friction torque caused by the friction between the cam contour and the vane tip were experimentally investigated and theoretically analysed[6)]. This paper presents the influence of the friction between the cam contour and the vane tip on the mechanical efficiency and the design concept on the relationship between the cam lift and the vane thickness for the vane pump with high mechanical efficiency.

2. Nomenclature

b	: Width of cam ring, rotor and vane
F_v	: Pushing force of vane
κ	: Coefficient for T_n defined by equation (6)
N	: Pump speed
p_d	: Delivery pressure
p_s	: Suction pressure
Δp	: Pressure difference between delivery- and suction-pressure ($p_d - p_s$)
R_1	: Small radius of cam ring
R_2	: Large radius of cam ring
T	: Driving torque of pump
T_0	: Friction torque independent of Δp
T_n	: Friction torque of vanes
T_{th}	: Ideal torque of pump
ΔT	: Total friction torque ($= T - T_{th}$)
V_{th}	: Theoretical displacement of pump
w	: Vane thickness
z	: Number of vanes
ε	: Parameter defined as $(R_2 - R_1)/w$
η_m	: Mechanical efficiency ($= T_{th}/T$)
η_{tc}	: Ultimate value of η_m defined by equation (12)
λ	: Coefficient of friction at vane top

3. Structure and Friction Torque of Vane Pump

3. 1 Structure of Vane Pump

Figure 1 shows a cross-sectional view of a balanced vane pump, composed of a cam ring with an elliptic inner bore, a rotor with a series of radially disposed vanes, two side plates located at both sides of the rotor and a shaft. The pump is so designed that both suction and delivery ports in the pump are diametrically opposed to provide complete balance of all internal radial forces. In this pump, the side plates have vane back pressure grooves at the sides facing the rotor to introduce the delivery pump pressure to all the bottoms of the vane slot of the rotor. During the pump operation, the vane is always pushed from the bottom by the delivery pressure and rotates with the loads imposed by the vane tip on the cam contour. The rotor is driven by a shaft through a very loose-fitting spline and the rotor and vanes rotate between two side plates with running clearance.

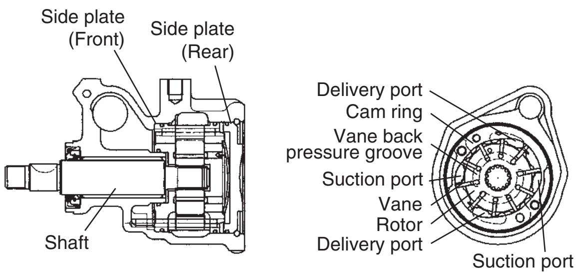


Fig. 1 Cross-sectional view of vane pump

Figure 2 shows notation of vane pump chamber used in this paper. The vane tip has roundness of radius R_v. **Figure 2** illustrates two pump chambers, partitioned by two vanes, in which small radius part (θ=0) and large radius part (θ=π/2) are located. The pump chamber (cross-hatched area) expands and oil is introduced into the chamber with pump rotation from θ=0 to π/2. And the pump rotation from π/2 to π causes the oil to be discharged from the pump chamber. The balanced vane pump will allow one more displacement process to take place between π and 2π, having a double displacement process, with two cycles of suction/discharge per revolution. Calculating the difference between the volume of the pump chamber at large radius part and that at small radius part and considering that the number of pump chambers is z, the pump displacement per revolution V_{th} is given by the following equation:

$$V_{th} = 2zb\left\{\frac{\pi}{z}(R_2^2 - R_1^2) - w(R_2 - R_1)\right\} \tag{1}$$

where z is the number of vanes, b is the width of the cam ring and the rotor, R_1 is the small radius of the cam ring, R_2 is the large radius of the cam ring and w is the vane thickness.

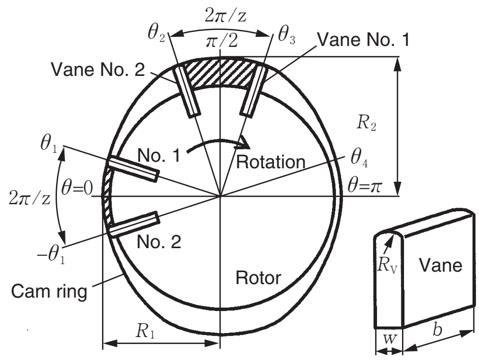


Fig. 2 Notation of vane pump chamber

Table 1 shows specifications of the test pumps including the cam lift and the vane thickness. Throughout the tests, each pump tested was assembled by new pump parts. The cam rings were made of ferro-sintered alloy and the inner surfaces of the cam rings were ground with roughness of 1-2 μm Rz. The vanes were finished by the barrel polishing and the roughness of the vane tip was about 0.3 μm Rz. The expression of roughness of Rz, average peak-to-valley heights, is the average value of the individual peak-to-valley heights in five continuous individual measurement sections. The hydraulic fluid used was commercial mineral oil and its density ρ and viscosity μ at 40℃ are 855 kg/m^3 and 0.0293 Pa·s, respectively.

Table 1 Specifications of vane pumps

Pump No.	A-1	A-2	B-1	B-2	B-3	C-1	C-2
Cam ring large radius R_2, mm	22.32	22.32	23.00	23.00	23.00	23.64	23.64
Cam lift $R_2 - R_1$, mm	2.32	2.32	3.00	3.00	3.00	3.64	3.64
Vane thickness w, mm	1.80	0.90	1.80	1.40	0.90	1.80	1.40
Vane tip radius R_v, mm	2.0	1.0	2.0	2.0	1.0	2.0	1.0
Pump displacement V_{th}, cm^3/rev.	7.94	8.56	10.45	10.81	11.25	12.84	13.28
ε = $(R_2 - R_1)$/w	1.29	2.58	1.67	2.14	3.33	2.02	2.60

3. 2 Experimental Results of Friction Torque Characteristics

The ideal pump driving torque T_{th} without energy loss in the pump is derived from the condition that the power required to drive the pump (the input) is equal to the hydraulic power (the output), as defined by the following equation:

$$T_{th} = \frac{V_{th}}{2\pi} \varDelta p \tag{2}$$

where $\varDelta p$ is the difference between the delivery pressure and the suction pressure.

Adding the friction torque in the vane pump $\varDelta T$ to the ideal torque, the actual pump driving torque T is expressed by the following equation:

$$T = T_{th} + \varDelta T \tag{3}$$

From equations (2) and (3), the following equation can be obtained:

$$\varDelta T = T - \frac{V_{th}}{2\pi} \varDelta p \tag{4}$$

First, the values of T against $\varDelta p$ for each test pump were experimentally measured, and $\varDelta T$ was calculated by using equation (4). In the experiment, T was measured with a torque meter and $\varDelta p$ was calculated by subtracting the suction pressure, p_s, measured at the pump inlet from the delivery pressure, p_d, measured at the pump outlet.

Three pumps (B-1, B-2 and B-3) with the cam ring of $R_2 - R_1$=3mm were used to investigate how three kinds of vane thickness affected the relationships between $\varDelta T$ and $\varDelta p$ at N=2 000 min^{-1} and oil temperature of 40℃.

Figure 3 shows the result of this investigation. $\varDelta T$ increases linearly with an increase in $\varDelta p$ and the slope is greater with an increase in vane thickness w. It is worthwhile to notice that all of the lines concentrate on one point at $\varDelta p$=0; namely, Y-intercept for all lines is identical. Drawing a line from this point as shown in the figure, $\varDelta T$ would be divided into T_0, a constant term independent of $\varDelta p$, and T_n, a term increasing linearly with an increase in $\varDelta p$. Then, $\varDelta T$ can be expressed by the following equation:

$$\varDelta T = T_0 + T_n \tag{5}$$

The friction torque in the vane pump is considered to be caused by (a) the friction between the shaft and the oil seal, (b) the friction between the shaft and the bearing, (c) the viscous friction between the rotor and the side plates and (d) the friction between the vane tip and the cam contour.

Factors (a) to (c) are considered to be independent of $\varDelta p$. This fact may be justified by the three following reasons.

First, a low pressure at a drain acts on the oil seal for the factor (a). Secondly, from the principle of a balanced vane pump, the force due to delivery pressure acts symmetrically on the rotor for the factor (b). Finally, the viscous friction due to shearing oil is independent of the pressure for the factor (c). In contrast, the friction torque due to the factor (d) increases with an increase in $\varDelta p$ because the force acting on the cam contour by the vane is proportional to $\varDelta p$, and that will be described in **Fig. 6** later. These reasons may prove that T_0 in equation (5) should constitute the friction torque due to factors (a) to (c) and T_n should be caused by the friction between the vane tip and the cam contour of the factor (d).

Figure 4 shows the relationship between w and T_n, which was arranged from the measured data at $\varDelta p$=5.88 MPa in **Fig. 3**. This figure shows that the straight line joining the measured data passes through the origin and T_n is directly proportional to w. From this fact that T_n is directly proportional to both w and $\varDelta p$, it is clear that the friction force between the vane tip and the cam contour is directly proportional to the vane force pushing the cam contour and it resembles the property of Coulomb's friction. As T_n is directly proportional to w and $\varDelta p$, T_n is expressed by the following equation:

$$T_n = \kappa w \varDelta p \tag{6}$$

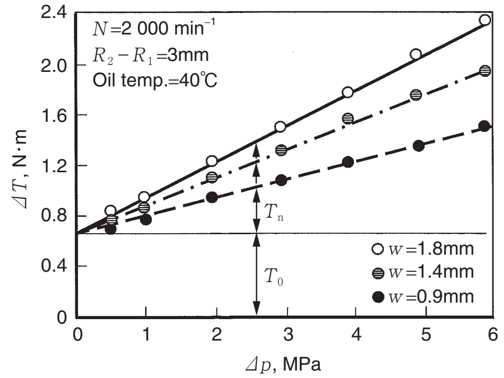


Fig. 3 Relationships between $\varDelta p$ and $\varDelta T$

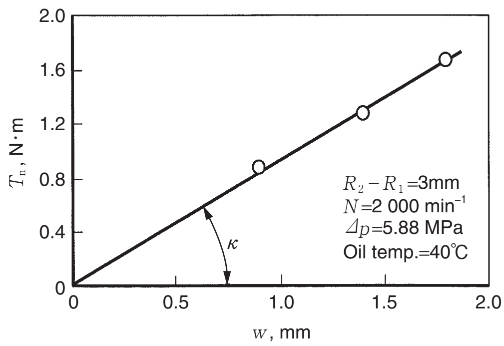


Fig. 4 Relationship between w and T_n

To investigate the dependence of κ on the pump speed N, the characteristics of $\varDelta T$ against $\varDelta p$ for three kinds of vane thickness were measured at various pump speeds, and **Fig. 5** shows the measured result of κ. As seen from this figure, κ decreases slightly with an increase in N. This fact suggests that the coefficient of friction between the vane tip and the cam contour reduces slightly with an increase in N and the distinctive difference in the vane thickness does not appear.

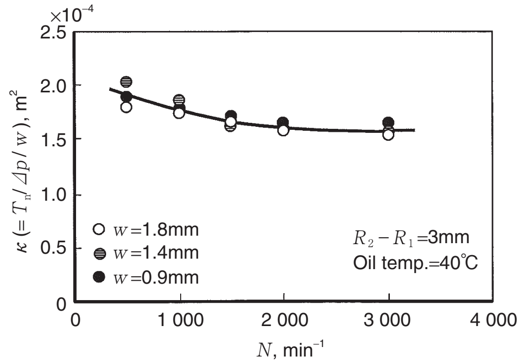


Fig. 5 Relationship between N and κ

4. Friction Torque at Vane Tip

4. 1 Theoretical Analysis on Friction Torque at Vane Tip

The upper part in **Fig. 6** illustrates the pressure acting on the bottom and the tip of the vane (e.g. vane No. 1 in **Fig. 2**) and the vane force F_v caused by the pressure difference in the small radius region ($0<\theta<\theta_1$), suction ($\theta_1<\theta<\theta_2$), large radius ($\theta_2<\theta<\theta_3$), and delivery ($\theta_3<\theta<\theta_4$). The delivery pressure p_d constantly acts on the vane bottom, and p_d as much as half the thickness of the vane acts on the vane tip at the small and large radius parts. Further, at the suction part, the suction pressure p_s acts on the entire vane tip. On the other side, at the delivery part, p_d acts also on the entire vane tip, balancing the radial force on the vane.

With the change of F_v taking place instantly at the starting point of each process, the changing pattern of F_v during one displacement process from θ=0 to π is illustrated in **Fig. 6**. In the case of this test pump, because the angle of suction part, θ_2-θ_1, is equal to that of delivery part, θ_4-θ_3, the average of F_v becomes $wb\varDelta p/2$. With the coefficient of friction between the vane tip and the cam contour regarded as λ, the friction force of the vane F_t is given as follows:

$$F_t = \frac{1}{2}\lambda wb\varDelta p \tag{7}$$

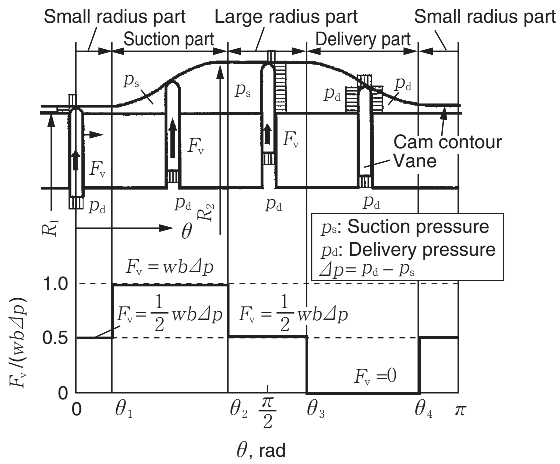


Fig. 6 Vane force during one displacement process

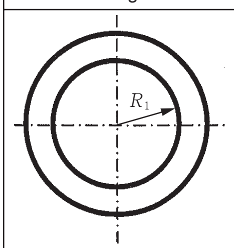
Seeking for a friction torque against a piece of vane by multiplying F_t by the average cam radius (R_2+R_1)/2 and considering that the number of vanes is z, the friction torque due to the friction between the vane tip and the cam contour T_n can be obtained by the following equation:

$$T_n = \frac{1}{4} z\lambda wb(R_2 + R_1)\varDelta p \tag{8}$$

4. 2 Measurement of the Coefficient of Friction at Vane Tip

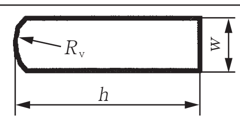
The coefficient of friction between the vane tip and the cam contour was experimentally measured by using a ring. The inner surface of the ring was ground with the same roughness as that of the cam rings for the test pump. The inner radius of the ring is the same as the small radius of cam ring. In this test, three kinds of vane thickness were used to investigate the influence of the vane thickness on the coefficient of friction. **Figure 7** shows the specifications of the test parts.

Ring



Inner radius: R_1 = 20mm
Width: b =15mm

Vane



(Unit: mm)

	①	②	③
Vane thickness: w	0.9	1.4	1.8
Vane height: h	8.7	8.7	8.7
Vane tip radius: R_v	1.0	2.0	2.0

Fig. 7 Specifications of test ring and vanes

Figure 8 shows a schematic diagram of the experimental apparatus and hydraulic circuit of the test for measuring the coefficient of friction. The apparatus is an actual pump equipped with a ring instead of a cam ring and the side plates without ports connected with pump delivery. Oil delivered from a feed pump was fed to the vane back pressure groove of the side plate in the test apparatus. The vanes in the test apparatus were lifted to touch their tips to the inner surface of the ring by the pressure regulated by a relief valve. All the vane tips ran on the inner surface of the ring under a pressure as low as zero. The shaft of the rotor in the test apparatus was driven by an electric motor via a torque meter.

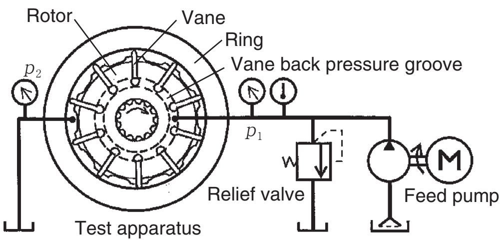


Fig. 8 Experimental apparatus and hydraulic circuit

Without the cam lift, the apparatus did not work as a pump to deliver oil, namely T_{th} expressed by equation (2) is zero. Therefore, the driving torque of this apparatus constitutes the total friction torque of the shaft, rotor and vanes. As described in **Fig. 3**, the friction torque of ten vanes, T_{n10}, could be measured by subtracting the friction torque of the shaft and rotor (corresponding to Y-intercept in **Fig. 3**) from the driving torque. The coefficient of friction at the vane tip λ could be calculated by the following equation:

$$\lambda = T_{n10}/(zbwR_1\varDelta p) \quad (9)$$

where z is the vane number, b the rotor width, w the vane thickness, R_1 the inner radius of the ring, and $\varDelta p$ the pressure difference between the inlet and outlet of the apparatus.

Figure 9 shows the relationship between a non-dimensional parameter S and the coefficient of friction between the vane tip and the ring inner surface λ, in which S is defined as the ratio to be obtained by diving the product of the viscosity of oil μ and the sliding speed of vane v by the vane pushing force a unit length F_v/b. In this figure, the values of λ only at $\varDelta p$=5.88 MPa for three kinds of w at various pump speeds N are plotted, and they seem to change along a curve. However, all of λ would not be on the curve for various combinations of $\varDelta p$ and N, even if the oil temperature is constant. Also, the non-dimensional parameter S is unpractical for the analysis in this study. Then, the change in λ against the rotational speed of the rotor was investigated.

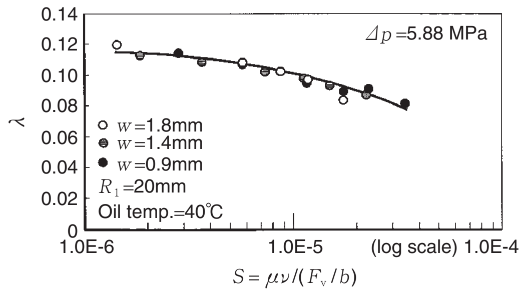


Fig. 9 Stribeck curve for coefficient of friction at vane tip

Figure 10 shows the relationship between N and λ for the vanes with three kinds of vane thickness. As seen from this figure, λ depends on the pump speed N and decreases gradually with an increase in N, namely the sliding speed of the vane tip. In this test, using the ring with the inner surface of 1-2 μm Rz roughness, λ ranges from 0.12 to 0.08 in the rotor speed region of 250-3 000 min^{-1}. The influence of vane thickness is slight and the relationship between N and λ could be indicated with a curve independently of w.

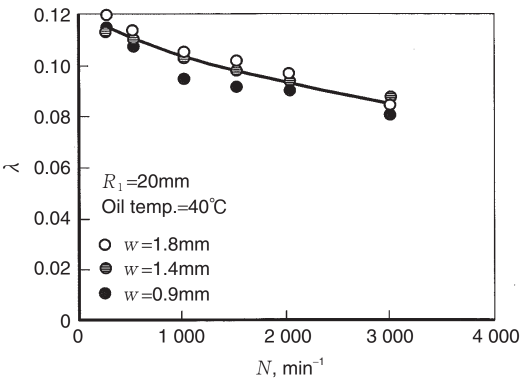


Fig. 10 Coefficient of friction at vane tip

Figure 11 shows the comparison between T_n obtained by the experiment and T_n calculated from equation (8). The calculation is performed by using λ=0.095 corresponding to the value at N=2 000 min^{-1} in **Fig. 10**. The value of T_n in the experiment almost coincides with the calculated one.

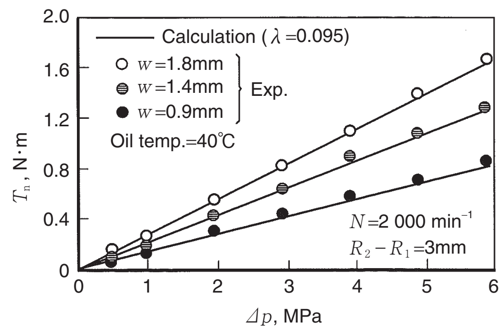


Fig. 11 Comparisons of T_n between calculation and experiment

5. Influence of Friction at Vane Tip on Mechanical Efficiency

5. 1 Theoretical Analysis on Mechanical Efficiency in Vane Pump

The mechanical efficiency η_m is defined as follows:

$$\eta_m = \frac{T_{th}}{T} \tag{10}$$

The values of η_m are calculated by using the measured pump driving torque T and the ideal torque T_{th} calculated from equation (2). As a sample, **Fig. 12** shows the relationship between Δp and η_m for the pump with w=1.4mm and cam lift (R_2–R_1)=3mm at N=2 000 min^{-1}. In this figure, η_m increases with an increase in Δp and then it would approximate to the value of η_{tc}.

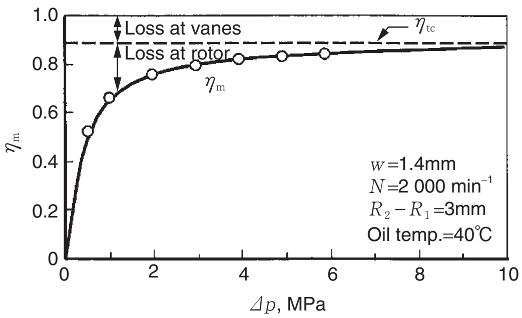


Fig. 12 Relationship between Δp and η_m

Further, by substituting equations (3) and (5) for equation (10), the following equation can be obtained:

$$\eta_m = \frac{T_{th}}{T_{th} + T_0 + T_n} \tag{11}$$

Then, η_{tc} in the following equation introduces an ultimate value of η_m.

$$\eta_{tc} = \lim_{\Delta p \to \infty} \eta_m = \lim_{\Delta p \to \infty} \frac{T_{th}}{T_{th} + T_0 + T_n} \tag{12}$$

Because both T_{th} and T_n are proportional to Δp and T_0 is independent of Δp, the ultimate value expressed as equation (12) results in the following equation:

$$\eta_{tc} = \frac{T_{th}}{T_{th} + T_n} \tag{13}$$

An asymptotic value of Δp-η_m curve is η_{tc}, and η_m becomes η_{tc} when Δp is extremely high. As seen from equations (11) and (13), the friction of the vane tip causes the mechanical efficiency of the vane pump to reduce from 1 to η_{tc}, and then the friction torque T_0 at the rotor sides, the shaft and the oil seal causes the mechanical efficiency to reduce from η_{tc} to η_m.

Substituting equation (8) for equation (13), η_{tc} becomes:

$$\eta_{tc} = \frac{T_{th}}{T_{th} + (1/4) z \lambda w b (R_2 + R_1) \Delta p} \tag{14}$$

Substituting equation (1) for T_{th} in equation (2) and substituting the result for equation (14), the following equation can be obtained:

$$\eta_{tc} = \cfrac{1}{1 + \cfrac{\lambda z w}{4(R_2 - R_1)\left\{1 - \cfrac{zw}{\pi(R_2 + R_1)}\right\}}} \tag{15}$$

In the case of the test pumps, because $zw/\{\pi(R_2+R_1)\}$ is much smaller than 1, this term can be eliminated. As a result, equation (15) can be expressed with a good accuracy of approximation by the following equation:

$$\eta_{tc} = \frac{1}{1 + \lambda z/(4\varepsilon)} \tag{16}$$

where $$\varepsilon = \frac{R_2 - R_1}{w} \tag{17}$$

Equation (16) suggests that the pumps with the same ε will have the same η_{tc} even if the pump dimensions such as cam lift and vane thickness are changed, and the larger value of ε as well as the reduction of λ and z may increase η_{tc}. In addition, equation (16) means that η_{tc} is independent of Δp.

Figure 13 shows the relationships between ε and η_{tc} for z=10, which is calculated from equation (16) for various λ. As seen from this figure, η_{tc} depends greatly on ε as well as λ and increases sharply with an increase in ε, when ε is less than 1. However, ε larger than 3 may not increase η_{tc} substantially, resulting in less contribution to improve η_{tc}. To achieve more 90 per cent of η_{tc}, λ should be reduced to about 0.1. Thus, reducing λ to less than 0.05 could bring the η_{tc} close to 95 per cent.

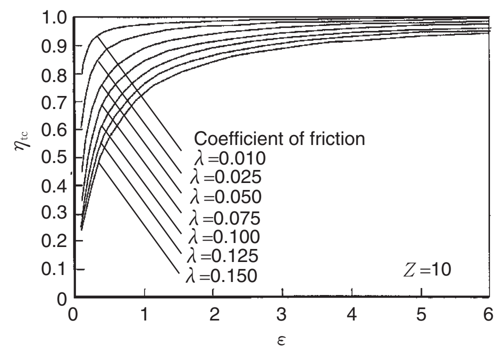


Fig. 13 Relationships between ε and η_{tc}

Figure 14 shows the relationship between λ and η_{tc} for various values of ε. This figure shows that η_{tc} is also affected greatly by λ.

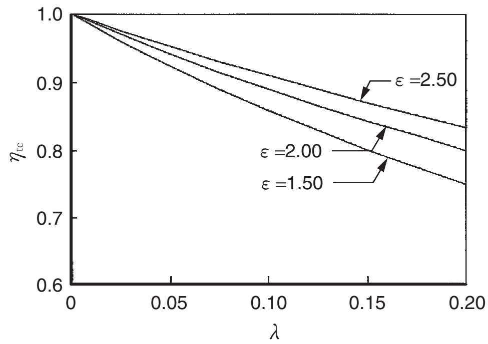


Fig. 14 Relationships between λ and η_{tc}

5. 2 Comparison of η_{tc} between Experiment and Calculation

For all the pumps shown in **Table 1**, the experimental values of η_{tc} were investigated. **Table 2** shows η_{tc} at four pump speeds and the values of ε for each test pump. The values of η_{tc} in **Table 2** are the average of η_{tc}, ranging from 1 to 5 MPa of Δp. Equation (16) suggests that η_{tc} is independent of Δp, and the fact has already been revealed experimentally[6].

Figure 15 shows the comparison of the experimental results of η_{tc} given in **Table 2** with the calculated ones. The values of λ used for the calculation were 0.125 and 0.1. **Figure 15** shows that a pump with a larger ε produces a higher η_{tc} and pump with the same ε results in almost the same η_{tc} despite variation of the vane thickness and the cam lift. Strictly speaking, the experimental points are plotted almost along the curve of the calculated values for λ=0.125 at N=1 000 min^{-1} and close to that for λ=0.1 at N=3 000 min^{-1}. It may be verified that the decrease of λ improves η_{tc} at a higher N.

Table 2 Estimated η_{tc} from experimental data

Pump No.	ε	η_{tc} (Oil temp.= 40℃) 1 000 min^{-1}	1 500 min^{-1}	2 000 min^{-1}	3 000 min^{-1}
A-1	1.289	0.822	0.827	0.840	0.846
A-2	2.578	0.907	0.910	0.917	0.915
B-1	1.667	0.843	0.849	0.863	0.847
B-2	2.143	0.867	0.886	0.886	0.888
B-3	3.333	0.921	0.924	0.926	0.922
C-1	2.022	0.865	0.876	0.883	0.893
C-2	2.600	0.893	0.900	0.908	0.917

The Author has revealed that the friction between the cam contour and the vane tip is reduced by reducing the surface roughness of the cam contour[7]. Then, by using the vane pumps with the specifications of the cam ring and the vane indicated as B-2 having ε of 2.14, η_{tc} was experimentally investigated with changing the surface roughness of the cam contour and the coefficient of friction at the vane tip. **Figure 16** shows the results, and the relationships between λ and η_{tc} obtained from the experiment agree well with the curve calculated from equation (16). As the pump speed N becomes higher, λ becomes lower and η_{tc} increases.

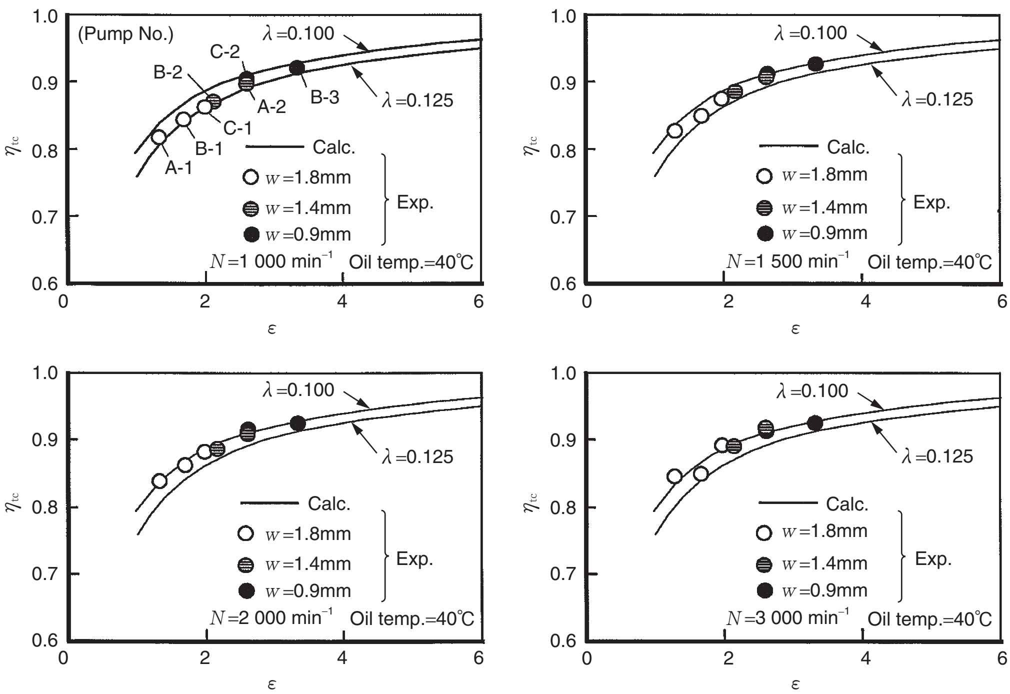


Fig. 15 Relationships between ε and η_{tc} (Experimental results)

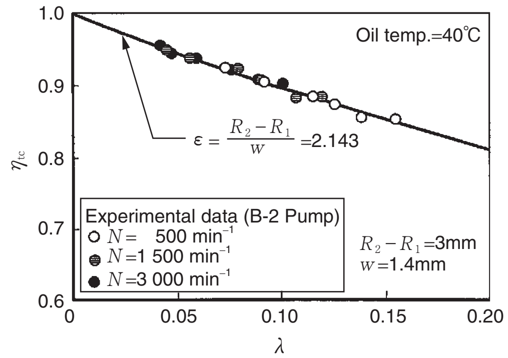


Fig. 16 Relationship between λ and η_{tc} (Experimental results)

6. Conclusions

In this study, the friction torque characteristics of the vane pump, especially the friction torque characteristic between the vane tip and the cam contour and its influence on the mechanical efficiency were experimentally and theoretically investigated. As a result, the following conclusions are obtained.

The friction force of the vane tip against the cam contour is proportional to the vane thickness and also to the pressure difference between pump outlet and inlet. In other words, the friction force is proportional to the vane force, which is proportional to the vane thickness and the pressure difference. This behavior is similar to Coulomb's law. In addition, the coefficient of friction decreases as the sliding speed of the vane is increased.

Although the load at the contact point of the vane tip on the surface of the cam contour increases with an increase in vane thickness, enlarging the cam lift in proportion to the vane thickness can prevent the mechanical efficiency from going down. The pumps produce the same mechanical efficiency when the parameter ε defined as the ratio calculated by dividing the cam lift by the vane thickness remains the same, even if the cam lift or the vane thickness is varied. The mechanical efficiency becomes higher with an increase in ε.

To design the vane pump with high mechanical efficiency, it is necessary to reduce the coefficient of friction on the sliding surface as well as to adopt a larger value of ε.

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