Research on Crowning Profile to Obtain Maximum Load Carrying Capacity for Roller Bearings

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Roller bearings, tapered roller bearings and rolling elements supported by line contact points can obtain the maximum load carrying capacity by modifying the crowning profile in order to uniformly distribute the damage of materials in the contact region.

Conventionally, it is considered that the crowning profile which has the uniform pressure distribution in the contact region is the best geometry. This type of crowning profile is called the Lundberg crowning profile. However the damage of materials concentrates in the subsurface of roller end, if the crowning profile is shaped according to the Lundberg profile. The uniform pressure distribution does not cause the uniform damage distribution of materials. The damage of materials is estimated by subsurface stress components.

Therefore the crowning profile should be optimised by considering subsurface stress components. The crowning profile which distributes the uniform damage of materials in the contact region to obtain the maximum load carrying capacity is developed in the Koyo Seiko Co., LTD. by heralding the world on the crowning profile.

Key Words*: roller bearing, crowning, contact, subsurface stress*

1. Introduction

Rollers are generally used for rolling elements of rolling bearings used under heavy loads in order to enhance load carrying capacity. These bearings include cylindrical roller bearings, tapered roller bearings and spherical roller bearings.

Crowning profiles are usually provided on rollers and on axial profiles of inner and outer rings in order to avoid concentration of contact pressure.

Conventionally, the crowning profile that does not allow contact pressure concentration in roller - raceway contact and furthermore distributes contact pressure uniformly in the longitudinal direction of the contact area (direction of rolling axis of rollers) has been considered to be the best profile[1]. Named after its developer, this crowning profile is called "Lundberg crowning profile."

Because the Lundberg profile provides the roller end surface with an infinite value, it has been pointed out as being impossible for actual machining. Johnson-Gohar corrected this so as to provide a finite value for amount of crowning on the roller end surface that has an infinite value with the Lundberg profile[2].

The concept of uniformly distributed contact pressure in the longitudinal direction of the contact area is used in recent research as well. Henryk-Bogdan points out that the Lundberg profile is applicable to spherical roller bearings.[3]

On the other hand, static and dynamic strength is required of rolling bearings. If the points in dispute are narrowed down to contact of rollers and raceways, static strength means strength to withstand indentations forming on the rollers and raceways, and dynamic strength is strength to resist metal fatigue damage called "rolling fatigue."

In order to anticipate the rolling bearings' ability to resist plastic deformation and fatigue life in the designing process, the authors believe that contact pressure given by the Lundberg profile, in other words applied external force should not be used as the criterion and the material's damage estimated by subsurface stress distribution should be used as the criterion.

As a result of evaluating subsurface stress and contact pressure between rollers and raceways of rolling bearings from this perspective, even if contact pressure are uniformly distributed in the longitudinal direction of the contact area, the damage of material concentrates on the end subsurfaces of the rollers. This means that plastic deformation occurs from the end subsurfaces of the rollers under the heavy loads. It also suggests that fatigue tends to be occured from the end subsurfaces of the rollers. In other words, the design concept expressed by Lundberg that attempts to determine the optimal crowning profile from applied external force does not give the maximum static or the maximum dynamic load carrying capacity to roller bearings.

This paper presents the study results of the crowning profile that theoretically has maximum load carrying capacity by introducing the new design concept for roller bearings.

2. Dynamic Model

Contact between a roller and a raceway of a rolling bearing is replaced by a finite width cylinder and a half space contact model such as shown in **Fig. 1**.

As for the coordinate axes, the rolling direction of the roller is the X axis, the rolling axis direction is the Y axis, and the direction perpendicular to the half space surface (X-Y plane) is the Z axis.

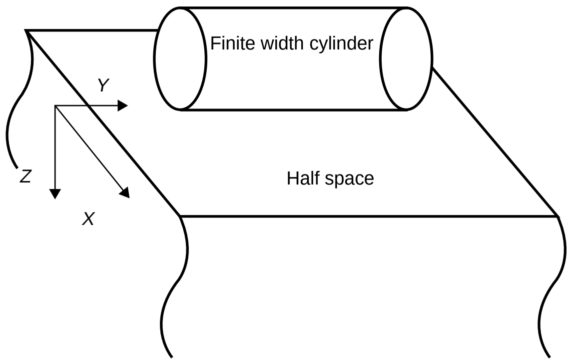

Fig. 1 3-dimensional contact model and coordinates

Details of the analyses are given in the following sections. The coordinate system of **Fig. 1** is used for analyses of both the dry contact problem and subsurface stress.

3. Numerical Analyses

The objective of numerical analyses in this paper is to determine the crowning profile that provides the rolling bearing with maximum load carrying capacity by uniformly distributing damage to the bearing material in the longitudinal direction of the contact area. In order to do so, numerical analyses consisting of two stages are conducted.

The first numerical analyses are analyses of dry contact between a roller and a raceway, and its objective is to determine dry contact pressure between a roller and a raceway. The second numerical analyses are analyses of damage of bearing material, in other words, analyses of subsurface stress.

3. 1 Three-dimensional Contact Pressure Analyses

Contact pressure between a roller of any crowing profile and a raceway cannot be determined from the classic Hertzian contact theory. The authors therefore decided to numerically solve the 3-dimensional dry contact problem.

The fundamental formulae for the dry contact problem can be expressed by two equations. The first equation is the formula for relative distance between two contacting bodies. Relative distance H is determined by the following equation:

$$H=\frac{hRx}{b^2}=\frac{h_0Rx}{b^2}=H_0+V \tag{1}$$

H_0 is the geometrical clearance between a roller and a raceway in non deformed contact, and V is the 3-dimensional elastic displacement expressed by the following equation:

$$V=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{P(X,Y)}{\sqrt{(X-X')^2+(Y-Y')^2}}\,dX'dY' \tag{2}$$

The second equation is the formula for force balance.

$$\frac{\pi L_{we}}{2b}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}P(X,Y)dXdY \tag{3}$$

When the sum of the amount of crowning formed on a roller and a raceway is defined as h_{cr}, crowning radius for coordinate y is determined by the following equation:

$$r_x=R_x-h_{cr} \tag{4}$$

Thus the distance between two contacting bodies taking crowning into account is determined by the following equation:

$$h_0=R_x-\sqrt{r_x^{\,2}-x^2} \tag{5}$$

The discrete equations are introduced from the fundamental formulae and contact pressure is solved by the NR method. Areas that do not originally contact become negative pressure, but analysis is continued by eliminating elements of negative pressure one by one until negative pressure no longer exists[4), 5)].

3. 2 Three-dimensional Analyses of Subsurface Stress

When contact pressure distribution is obtained, 3-dimensional subsurface stress distribution can be determined by numerically integration with the pressure distributions. The components of subsurface stress are replaced to the non-dimensional parameters in order to reduce the number of variables for analyses.

The various components of 3-dimensional subsurface stress of the Cartesian coordinates system are determined by the following formulae[6)]:

$$\Sigma_X=\frac{\sigma_x}{P_h}=\sum_{i=1}^{n_X}\sum_{j=1}^{n_Y}\frac{Pij}{2\pi}\left[\frac{1-2\upsilon}{\bar{r}^2}\left\{\left(1-\frac{Z}{\bar{R}}\right)\frac{(X-X'_i)^2-(Y-Y'_j)^2}{\bar{r}^2}+\frac{Z(Y-Y'_j)^2}{\bar{R}^3}\right\}-\frac{3Z(X-X'_i)^2}{\bar{R}^5}\right] \tag{6}$$

$$\Sigma_Y=\frac{\sigma_y}{P_h}=\sum_{i=1}^{n_X}\sum_{j=1}^{n_Y}\frac{Pij}{2\pi}\left[\frac{1-2\upsilon}{\bar{r}^2}\left\{\left(1-\frac{Z}{\bar{R}}\right)\frac{(Y-Y'_j)^2-(X-X'_i)^2}{\bar{r}^2}+\frac{Z(X-X'_i)^2}{\bar{R}^3}\right\}-\frac{3Z(Y-Y'_j)^2}{\bar{R}^5}\right] \tag{7}$$

$$\Sigma_Z=\frac{\sigma_z}{P_h}=\sum_{i=1}^{n_X}\sum_{j=1}^{n_Y}-\frac{3Pij}{2\pi}\frac{Z^3}{\bar{R}^5} \tag{8}$$

$$T_{xy}=\frac{\tau_{xy}}{P_h}=\sum_{i=1}^{n_X}\sum_{j=1}^{n_Y}\frac{Pij}{2\pi} \tag{9}$$

$$T_{yz}=\frac{\tau_{yz}}{P_h}=\sum_{i=1}^{n_X}\sum_{j=1}^{n_Y}-\frac{3Pij}{2\pi}\frac{(Y-Y'_j)Z^2}{\bar{R}^5} \tag{10}$$

$$T_{zx}=\frac{\tau_{zx}}{P_h}=\sum_{i=1}^{n_X}\sum_{j=1}^{n_Y}-\frac{3Pij}{2\pi}\frac{(x-X'_i)z^2}{\bar{R}^5} \tag{11}$$

And here:

$$\bar{r}^2=(X-X'_i)^2+(Y-Y'_j)^2 \tag{12}$$

$$\bar{R}^2=(X-X'_i)^2+(Y-Y'_j)^2+Z^2 \tag{13}$$

The objective of this paper is to numerically evaluate the damage of bearing material, but damage of bearing material by these stress components cannot be evaluated. In this paper, therefore, the damage of bearing material is evaluated by the equivalent stress used for determining von Mises' yield criterion. The equivalent stress shown as follows is given by the stress components of the cartesial coordinates system.

$$\bar{\sigma}=\sqrt{\frac{1}{2}\{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6\tau_{xy}^{\,2}+6\tau_{yz}^{\,2}+6\tau_{zx}^{\,2}} \tag{14}$$

4. Results and Discussion

4. 1 Lundberg Profile

In line contact, as the Hertzian elastic contact theory is not provide the relative approach between two contacting bodies, Lundberg attempted to overcome this problem. At first, Lundberg introduced the elastic displacement when an elliptical distributed load with finite width shown in **Fig. 1** acts on a half space, and use the elastic displacement equivalent to approach between the two bodies instead of the approximate valve of relative approach between two contacting bodies in line contact. Lundberg furthermore found that uniform contact pressure distribution shown in **Fig. 2** could be obtained when the elastic displacement curve given by the process of this research is used as the crowning curve between a roller and a raceway.

If crowning is not formed on a roller bearing, the contact pressure concentrates at the edges of the contact area between rollers and raceways. This would cause fracture from the ends of the rollers even under light loads. Lundberg therefore naturally thought he would develop a crowning profile that eliminated edge load, which would cause early fatigue.

Thus, the crowning profile that uniformly distributes contact pressure is Lundberg's design concept for bearings and is provided as following equation:

$$h_{CR}(y)=\frac{2F}{\pi E' L_{we}}\ln\left[\left\{1-\left(\frac{2y}{L_{we}}\right)^2\right\}^{-1}\right] \tag{15}$$

With this crowning curve, the amount of crowning at the center of a roller is "0", and that at the ends of the roller is infinity. The notion of crowning which is larger than the radius of the rollers is physically falseness.

Johnson-Gohar therefore introduced the following modified formula to eliminate the singular point whereby the amount of crowning is infinite.

$$h_{CR}(y)=\frac{2F}{\pi E' L_{we}}\ln\left[\left\{1-\left(1-0.3033\frac{2b}{L_{we}}\right)\left(\frac{2y}{L_{we}}\right)^2\right\}^{-1}\right] \tag{16}$$

Equations (15) and (16) are introduced for the purpose of uniformly distributing contact pressure. It is believed that if the equations are applied to crowning profiles of roller bearings, uniform contact pressure distribution in the longitudinal direction of the contact area can be obtained.

However, if the equations are actually applied to crowning profiles of the rollers and raceways and contact pressure is analyzed, a pressure peak at the ends of the rollers shown in **Fig. 3** can be seen. The cause of this pressure peak has still not been discussed, so it is decided to study it here.

Even though the Lundberg profile was used as the generatrix profile, the peak pressure rising is caused by the assumption that contact width is constant. In other words, by machining crowning on the rollers and raceways, the radius of rollers of the part machined with crowning should become smaller, and the contact width of the ends of the rollers should also be reduced.

When Lundberg introduced the amount of elastic displacement, however, the assumption that contact width was constant was used. Contact pressure per unit width therefore increased with the radical reduction of contact width at the ends of the rollers.

To be exact, therefore, the Lundberg profile is not the crowning profile that uniformly distributes contact pressure in the longitudinal direction of the contact area.

Furthermore, even if it is assumed that the uniform pressure in the longitudinal direction of the contact area is loaded as shown in **Fig. 2**, equivalent stress is concentrated at the ends of the roller as shown in **Fig. 4**.

Even if contact pressure were to be uniformly distributed in the longitudinal direction of the contact area, plastic deformation occurs in the ends of the roller, so the bearing is not given the maximum load carrying capacity.

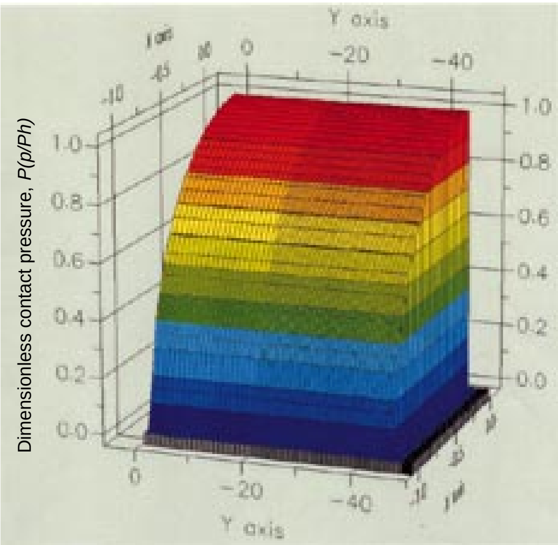

Fig. 2 Lundberg's ideal contact pressure distribution profile between roller and raceway

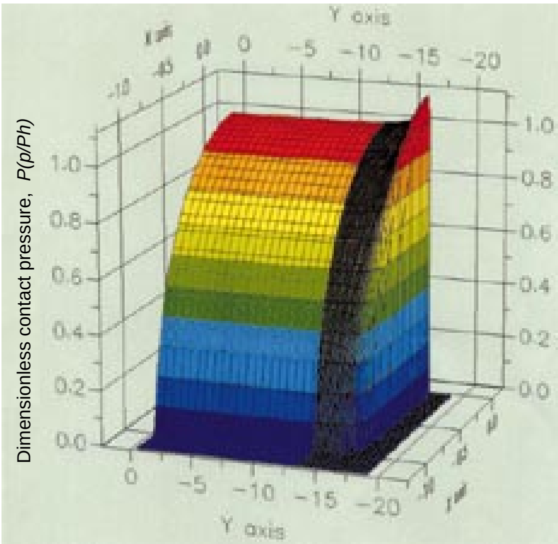

Fig. 3 Contact pressure between roller and raceway machined according to Lundberg's profile

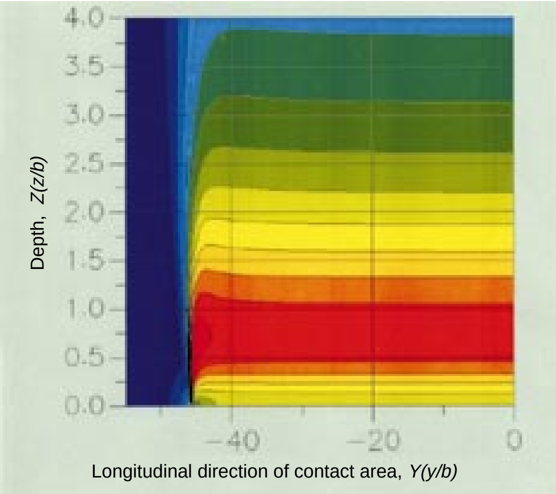

Fig. 4 Equivalent stress distribution under uniformly distributed contact pressure in longitudial direction

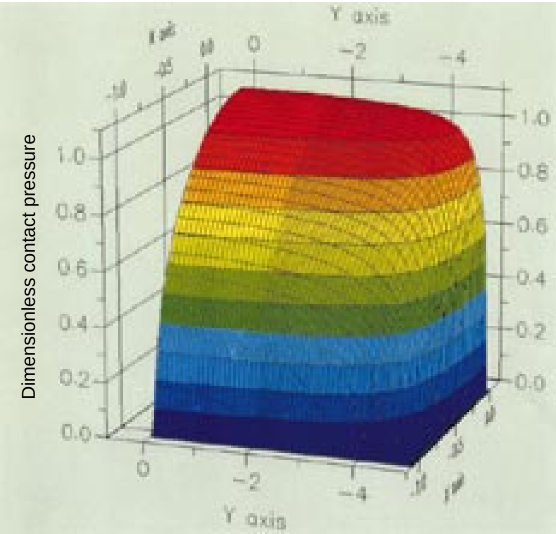

Fig. 5 Result of contact pressure distribution analysis (crowning profile to obtain maximum load carrying capacity, L_{we}/b=10)

4. 2 Crowning Profile to Obtain Maximum Load Carrying Capacity

In order to obtain the maximum load carrying capacity in theory for the roller bearings, the damage of bearing materials must be uniformly distributed in the longitudinal direction of the contact area.

This is the new design concept proposed in this paper. The numerical analyses of contact pressure and subsurface stress are conducted, and a crowning profile that satisfied the design concept is found.

The distributions of contact pressure between the roller and the raceway are shown in **Figs. 5 ～ 7**, and equivalent stress distributions are shown in **Figs. 8 ～ 10**. In these figures, dimensionless effective length of the roller is L_{wc}/b = 10, L_{wc}/b = 100 and L_{wc}/b = 1 000 respectively.

In **Figs. 5 ～ 7**, the vertical axis is the dimensionless contact pressure P which is non dimensionalized by the maximum Hertzian contact pressure, and the horizontal axis is the dimensionless X and Y coordinates non-dimensionalized by the Hertzian contact half-width b. Distribution of contact pressure is axisymmetric to the X and Y axes, so only 1/4 of the elements are explained. As Lundberg estimated, contact pressure is not uniform, but rather gradually decreases at the ends of the roller. The degree of contact pressure decreasing at the ends of the roller differs according to the aspect ratio (ratio of effective length of the roller to Hertzian contact half width).

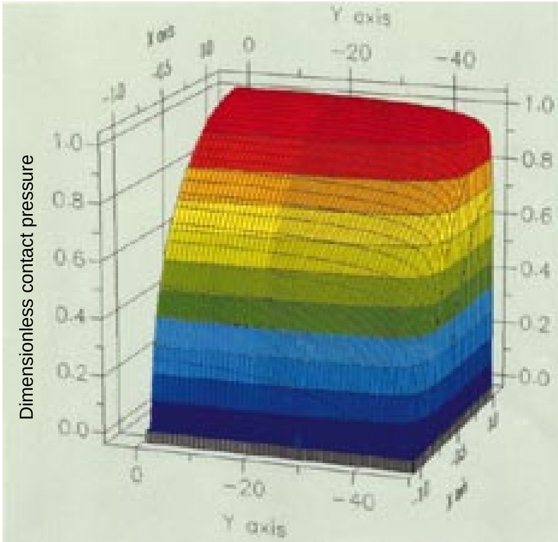

Fig. 6 Result of contact pressure distribution analysis (crowning profile to obtain maximum load carrying capacity, L_{we}/b=100)

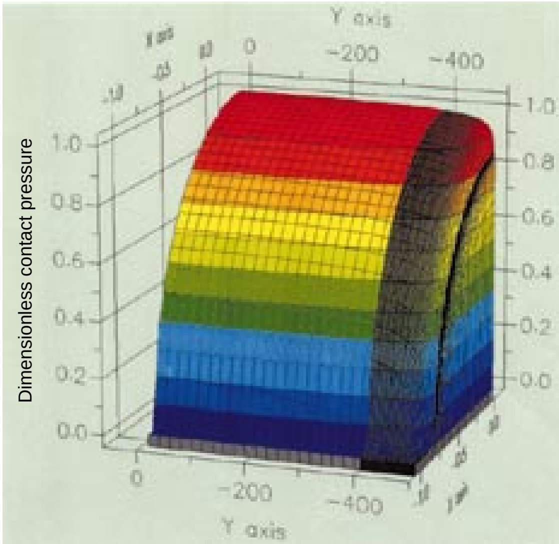

Fig. 7 Result of contact pressure distribution analysis (crowning profile to obtain maximum load carrying capacity, L_{we}/b=1 000)

Figures 8 ～ 10 show equivalent stress as examples of subsurface stress components that numerically estimates damage of materials. The vertical axis is depth and the surface is the position of zero. This subsurface stress is located below the X axis coordinate. Because distribution of subsurface stress is axisymmetrical to the coordinate axis Y = 0, only 1/2 elements are shown. The value of equivalent stress becomes larger as the color becomes red.

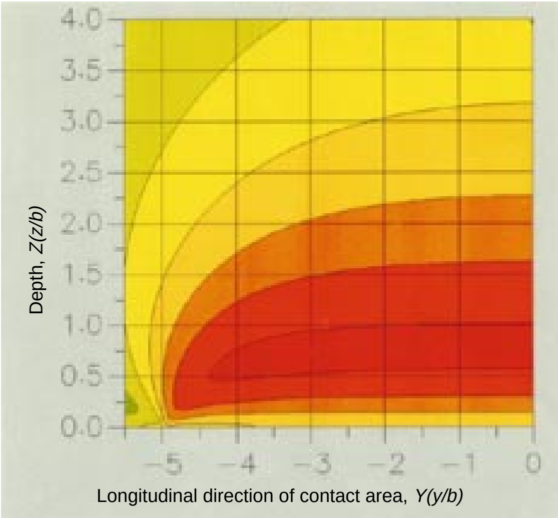

Fig. 8 Result of equivalent stress distribution analysis (crowning profile to obtain maximum load carrying capacity, L_{we}/b =10, X= 0)

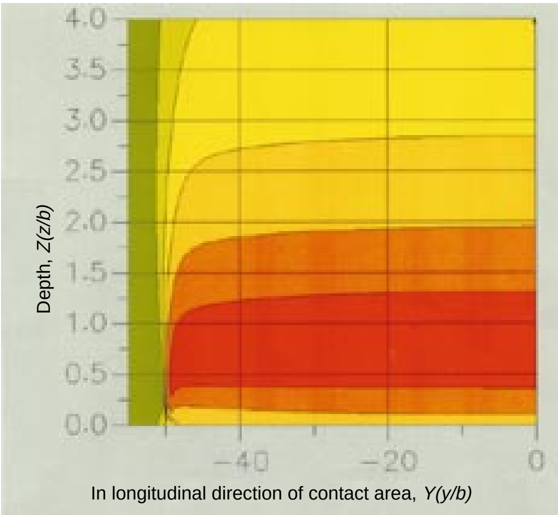

Fig. 9 Result of equivalent stress distribution analysis (crowning profile to obtain maximum load carrying capacity, L_{we}/b =100, X= 0)

Figures 8 ～ 10 show the position where equivalent stress becomes largest is the depth of 0.7 b ～ 0.8 b. The figures also show that the value of equivalent stress is uniform in the longitudinal direction of the contact area and damage of the bearing material is also uniform.

Therefore, plastic deformation is not produced until the materiul's strength limit exceeded by the damage caused by external force, and the crowning profile theoretically has maximum resistance to indentation.

Because the damage of the bearing material is uniform, the dynamic fatigue strength improving is expected.

If the strength of bearing material depend on von Mises' yield criterion, the crowning profile that provides the roller bearing with maximum load carrying capacity can be determined by the following equation:

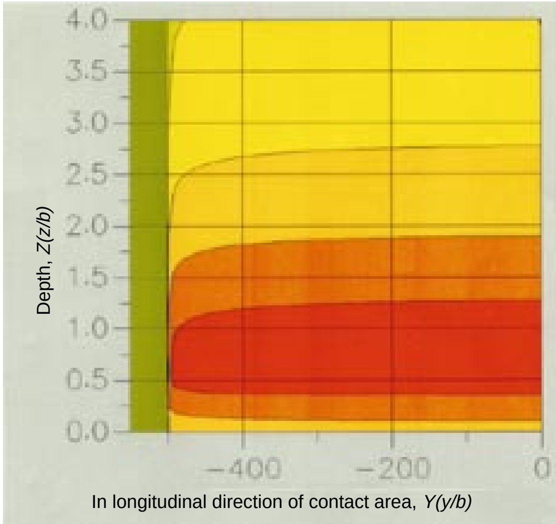

Fig. 10 Result of equivalent stress distribution analysis (crowning profile to obtain maximum load carrying capacity, L_{we}/b =1 000, X= 0)

$$h_{cr}(y) = 4Rk_2\left[\frac{\sigma_E}{0.557E'}\right]^2 \ln\left\{\frac{1}{1-(2k_1 y/L_{we})^2}\right\} \tag{17}$$

Here σ_E is tensile yield stress (compressive yield stress).

If the strength of bearing material depend on Tresca's yield criterion, the equation is as follows:

$$h_{cr}(y) = 4Rk_2\left[\frac{\tau_{max}}{0.3E'}\right]^2 \ln\left\{\frac{1}{1-(2k_1 y/L_{we})^2}\right\} \tag{18}$$

τ_{max} is shearing yield stress.

The following equation may be more useful for bearing design:

$$h_{cr}(y) = \frac{2k_2 F}{\pi E' L_{we}} \ln\left\{\frac{1}{1-(2k_1 y/L_{we})^2}\right\} \tag{19}$$

Equation (19) is the equation for obtaining optimal design from rolling element load. Rolling element load F must not exceed the rolling element load of basic static load rating.

Here:

$$k_1 = \sqrt{1-\frac{1}{\exp\left[\frac{1}{0.2717+0.4783\left(\sqrt{L_{we}/b}\right)^{-1}}\left\{0.2501\ln(L_{we}/b)+0.4725\right\}\right]}} \tag{20}$$

$$k_2 = 1.25-2.2\left(\sqrt{L_{we}/b}\right)^{-1} \tag{21}$$

The k_1 can be approximated as 1, but the amount of crowning at the roller ends becomes infinite.

5. Conclusions

By investigating the contact between rollers and raceways of rolling bearings, the numerical analyses of contact pressure and subsurface stress are condocted, and the following conclusions are given:

1. Even if the crowning profiles between rollers and raceways of rolling bearings are machined to obtain the contact pressure distribution that Lundberg aimed for, plastic deformation is caused from the ends of the rollers under heavy loads. In Lundberg's design concept, therefore, roller bearings do not give the maximum load carrying capacity.
2. The study of subsurface stress under contact of rollers and raceways indicates a design concept for rolling bearings with maximum load carrying capacity that theoretically uniformly distributes damage of materials in the longitudinal direction of the contact area.
3. The simple formula to desigh the crowning profile with the maximum static and dynamic load carrying capacity under any usage conditions are introduced.

<Symbols>

b : Half of Hertzian contact width for line contact, x axis direction (m)

$$b = Rx\sqrt{\frac{8W}{\pi}}$$

E_1, E_2 : Young's modulus of bodies 1 and 2 (N/m^2)

E' : Equivalent Young's modulus (N/m^2)

$$\frac{1}{E'} = \frac{1}{2}\left[\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}\right]$$

F : Rolling element load (N)

h_0 : Clearance between a roller and a raceway in the non-deformed contacting condition (m)

h_{cr} : Amount of crowning (m)

i, j : Numbers of discreted elements

k_1 : Coefficient

k_2 : Coefficient

L_{we} : Effective roller length (m)

n_x, n_y : Number of discreted contact pressure elements (x direction, y direction)

P_h : Maximum Hertzian contact pressure (Pa) of line contact theory

$$P_h = \frac{E'b}{4Rx} = \frac{2w}{\pi b} = E'\sqrt{\frac{W}{2\pi}}$$

$p_{i,j}$: Contact pressure (Pa) for discreted coordinates (x_i y_j)

$P_{i,j}$: Dimensionless contact pressure for discreted coordinates (x_i y_j)

r_{x1}, r_{x2} : Main curve radius of contact region in the x direction for bodies 1 and 2 (m)

r_x : Roller radius for x (m)

R_x : Equivalent radius in x direction, max. roller diameter (m)

$$\frac{1}{Rx} = \frac{1}{r_{x1}} + \frac{1}{r_{x2}}$$

w : Load per unit width in y axis direction, $w = F/L_{we}$ (N/m)

W : Load parameter, $W = w/(E'Rx)$

x, y, z : Coordinates (m)

X, Y, Z : Dimensionless coordinates (points of calculating stress), $X = x/b$, $Y = y/b$, $Z = z/b$

x', y' : Points where pressure acts (m)

X', Y' : Dimensionless coordinates (points where pressure acts), $X' = x'/b$, $Y' = y'/b$

υ_1, υ_2 : Poisson's ratio for bodies 1 and 2

π : Ratio of the circumference of the circle to its diameter

σ_x : Compressive stress component that acts perpendicularly to any point on the yz plane inside the material (Pa)

σ_y : Compressive stress component that acts perpendicularly to any point on the xz plane inside the material (Pa)

σ_z : Compressive stress component that acts perpendicularly to any point on the xy plane inside the material (Pa)

τ_{xy} : Shearing stress component that acts parallel to any point on the xy plane inside the material (Pa)

τ_{yz} : Shearing stress component that acts parallel to any point on the yz plane inside the material (Pa)

τ_{zx} : Shearing stress component that acts parallel to any point on the zx plane inside the material (Pa)

τ_{MAX} : Maximum shearing stress component at any point inside the material (Pa)

σ_{E} : Equivalent stress component that acts on any point inside the material (Pa), used for determination of von Mises' yield criterion (Pa)

Σ_X : Dimensionless compressive stress that acts perpendicularly to any point on the yz plane inside the material, $\Sigma_X = \sigma_x / P_h$

Σ_y : Dimensionless compressive stress that acts perpendicularly to any point on the xz plane inside the material, $\Sigma_y = \sigma_y / P_h$

Σ_z : Dimensionless compressive stress that acts perpendicularly to any point on the xy plane inside the material, $\Sigma_z = \sigma_z / P_h$

T_{xy} : Dimensionless shearing stress that acts parallel to any point on the xy plane inside the material, $T_{xy} = \tau_{xy} / P_h$

T_{yz} : Dimensionless shearing stress that acts parallel to any point on the yz plane inside the material, $T_{yz} = \tau_{yz} / P_h$

T_{zx} : Dimensionless shearing stress that acts parallel to any point on the zx plane inside the material, $T_{zx} = \tau_{zx} / P_h$

T_{MAX} : Maximum dimensionless shear stress at any point inside the material, $T_{\mathrm{MAX}} = \tau_{\mathrm{MAX}} / P_h$

Σ_{E} : Dimensionless equivalent stress component that acts on any point inside the material (Pa), used for determination of von Mises' yield criterion, $\Sigma_{\mathrm{E}} = \sigma_{\mathrm{E}} / P_h$

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